SCHOOL OF BUSINESS ADMINISTRATION
WORKING PAPER SERIES

THE INFORMATION CONTENT OF IMPLIED PRICES: TEST OF THE OPTION BOUNDARY APPROACH

Peter Lung
University of Dayton

R. Stephen Sears
Texas Tech University

WP03-11F

The information content of implied prices: A test of the option boundary approach

Peter Lung*
University of Dayton

R. Stephen Sears
Texas Tech University

Abstract: This paper examines the information content of implied prices derived from option markets. Because of non-synchronous trading, option model misspecification, early exercise effect, and market frictions, previous literature documents mixed results on the predictability of implied prices. To avoid these hindrances, this paper provides a new methodology, the option boundary approach, to extract implied prices based upon option boundaries. The implied prices derived based on the approach are free from the effects of model misspecification, early exercise, and market frictions. The empirical evidence indicates that there is information content in option markets. After ARMA-TARCH effects on price changes are removed, the test results are still significant.

Classification Code: G14

Keywords: Options Boundaries, Implies prices.

Authors are listed in alphabetical order. * Contact author: Peter P. Lung, Department of Economics and Finance, University of Dayton, Dayton, OH 45469-2251. Tel: (937)-229-3095, Fax: (937)-229-2477, E-mail: Peter.Lung@notes.udayton.edu.

The information content of implied prices: A test of the option boundary approach

Abstract

This paper examines the information content of implied prices derived from option markets. Because of non-synchronous trading, option model misspecification, early exercise effect, and market frictions, previous literature documents mixed results on the predictability of implied prices. To avoid these hindrances, this paper provides a new methodology, the option boundary approach, to extract implied prices based upon option boundaries. The implied prices derived based on the approach are free from the effects of model misspecification, early exercise, and market frictions. The empirical evidence indicates that there is information content in option markets. After ARMA-TARCH effects on price changes are removed, the test results are still significant.

Introduction

Literature has documented mixed results about the information content of implied prices in option markets. Manaster and Rendleman (1982), Tucker (1987), and Finucane (1991) among others provide various methodologies to extract expected returns from option markets and evidence predictability in option premiums. Kumar, Sarin, and Shastri (1992) also show that abnormal option returns lead block trades in the underlying stock for about half an hour. More recently, based upon stock options over 1988 to 1992, Chakravarty, Gulen, and Mayhew (2002) document 12% to 23% price discovery occurring in the option market. Contrarily, using high frequency option transaction data, Stephan and Whaley (1990), Pan, Hocking, and Rim (1996), and Brenner, Eom, and Landskroner (1996) among others find that option markets do not reveal information beyond that already observed in the underlying assets' markets. Bhattacharya (1987), Chan, Chung, and Johnson (1993), Diltz and Kim (1996), O'Connor (1999), and Chan, Chung, and Fong (2002) also indicate littler or no evidence that option markets lead stock markets.

Existing research has focused on the examination of information flow, cause-effect relationship, and price discovery, while ignored the methodology of extracting the potential information in option markets. When the derivation of implied prices suffers from non-synchronous trading, option model misspecification, early exercise effect, and market frictions, the empirical evidence would be debatable. With this in mind, this study employs option boundaries to construct implied prices and examines the relationship between the implied and realized underlying asset's prices. This approach not only avoids model misspecification, but also incorporates market frictions and early exercise effect into the construction of the implied prices.

The option boundary approach provided in this study requires intensive intraday option data which shall have synchronous bid, ask, and transaction prices for both underlying assets and options. The tick-by-tick currency option data provided by the Philadelphia Stock Exchange (hereafter PHLX) is employed. The empirical evidence suggests that there is information content in option markets. Market frictions, early exercise effect, and the option boundary violations are not serious hindrance to the information content of the implied prices derived based on the option boundary approach. It is also worth noting that option boundary violations seem related to information trading. The test results are robust after ARMA-TARCH effects are considered.

The remainder of this paper is organized as follows. Section I reviews previous literature. Section II discusses the option boundary approach that is employed to extract the potential information contained in option premiums. Section III describes sample data. Section IV presents hypotheses and empirical results. Section V concludes.

I. Literature Review

Previous studies have derived implied prices from option premiums in three different methods—(1) the joint-estimation approach, (2) the sequential approach, and (3) the put-call parity approach. In order to extract the option market's assessment of the underlying asset's price, while at the same time avoiding the difficulties associated with implied volatility measurement error, the joint-estimation approach (Manaster and Rendelman (1982), Bhattacharya (1987), and Tucker (1987)) simultaneously estimates the implied price and implied volatility based upon an option pricing model, given all other observed variables besides underlying asst's price and volatility. Alternatively, the sequential approach (Stephan and Whaley (1990), Pan, Hocking, and Rim (1996), Brenner, Eom, and Landskroner (1996), and Chakravarty, Gulen, and Mayhew (2002)) inserts the previous period's implied volatility into an option model and solves for the implied price over the current period. On the other hand, through the put-call parity relationship, the put-call parity approach (Finucane (1991)) expresses the implied price as a linear function of call, put, and strike prices, and derives the implied price based on the function.

Employing option prices as predictors of equilibrium prices was first introduced by Manaster and Rendleman (1982). Their joint-estimation approach suggests that options are actually priced according to an option pricing model. If the model is correct, then the implied price would be the option market's assessment of the equilibrium value of underlying asset. Using daily stock option data from 1973 through 1976, they find that implied prices contain information that is not fully reflected in observed stock prices for a period of up to 24 hours. This evidence is also confirmed by Tucker (1987) who applies the same approach to currency forwards and option markets. Contrarily, considering non-synchronous trading problem, Stephan and Whaley (1990) use the sequential approach and employ intraday trading records in the stock and stock option markets during the first quarter of 1986 to examine the lead-lag relation between the implied and market prices. Their results indicate that the information contained in implied stock prices cannot predict future stock price changes. According to the same approach, Brenner, Eom, and Landskroner (1996) and, Hocking, and Rim (1996) also find that currency option market cannot reflect more information than the spot market.

Because of the potential wrong-model problem in the studies above, Finucane (1991) provides the put-call parity approach to derive the implied price from option premiums and evidences the information content in implied prices based on the intraday OEX index options over the period from 1985 through 1988. Strictly speaking, the put-call parity can be applied only to European options in a frictionless market. Because OEX options are American style and markets are not frictionless, early exercise effect and market frictions may distort the estimates for implied prices.

In summary, the estimates for implied prices contain two components—the potential information component and the measurement error. The sources of measurement errors are non-synchronous trading records, model misspecification, market friction, and early exercise effect. Therefore, the empirical findings about information content in implied prices are inconclusive.

II. The Option Boundary Approach

II. 1 Construction of the divergences between implied and market prices

Because the previous methodologies of extracting implied prices are insufficient when option models are misspecified and early exercise effect exist, this paper provides the option boundary approach to derive the divergences between implied and market prices and use the divergences to examine the information content in option markets. When market is frictionless, the boundary conditions for American call and put options can be obtained using the Law of Arbitrage. The boundaries are shown in (1) and (2):

$$S^0 + P - X e^{-rd \times \tau} \geq C \geq S^0 - X \quad (1)$$

$$X + C - S^0 e^{-rf \times \tau} \geq P \geq X - S^0 \quad (2)$$

where C (P) is call (put) option premium, S_0 is underlying asset's price, rd is the risk-free rate, rf is annualized dividend yield,[1] X is exercise price, τ is time to maturity, and e denotes exponential function. The left-hand sides of these two inequalities indicate that a portfolio consisting of: (1) the value of the spot price (or exercise price) plus (2) a put (or a call), and minus (3) the present value of exercise price (or spot price), shall be no less than the value of a call (or a put). This should hold because the payoff of the left-hand side portfolio is always no less than that of a call (or a put) in all future states. The right hand sides of inequalities imply that an American call (put) is no less than the exercise value. Thus, the upper boundary violation for a call can be measured as:

$$VIO_{CU} = C - S^0 + P - X e^{-rd \times \tau} \quad (3)$$

A positive VIO_{CU} would indicate that the upper boundary for a call option is violated and the call is relatively more expensive (overpriced) than the corresponding put. The larger the magnitude of VIO_{CU} is, the greater the degree of violation. Similarly, the degree of violation for the upper boundary of a put option is expressed as:

$$VIO_{PU} = P - X + C - S^0 e^{-rf \times \tau} \quad (4)$$

[1] For currency options, rd and rf are the domestic and foreign risk-free rates, respectively.

A positive VIO_{PU} implies that the upper boundary for a put option is violated. Based on equations (3) and (4), the divergence between the implied and market prices is constructed as:

$$
\begin{aligned}
DIV \quad &= VIO_{CU} - VIO_{CU} \\
&= [C - (S^0 + P - X\, e^{-rd \times \tau})] - [P - (X + C - S^0 e^{-rf \times \tau})] \\
&= 2(C-P) + (X\, e^{-rd \times \tau} - X) - (S^0 - S^0 e^{-rf \times \tau}) \qquad (5)
\end{aligned}
$$

A positive (negative) DIV signals that the range estimate of implied price is biased to the upside (downside) of the market price and indicates that the price is more likely to increase (decrease), if here is information content in option markets

In the presence of transaction costs and bid-ask spreads, the upper boundary conditions for American call and put options can be specified as:

$$(P^a + S_0^a - X \cdot e^{-rd,a \cdot \tau}) + (T_{X,P} + T_S + T_P) \geq C^a + T_C \geq C^a - T_C \geq C^b - T_C \qquad (6)$$

$$(C^a - S_0^b \cdot e^{-rf,a \cdot \tau} + X) + (T_{X,C} + T_S + T_C) \geq P^a + T_P \geq P^a - T_P \geq P^b - T_P. \qquad (7)$$

where the upper cases, "^{a}" and "^{b}", stand for the ask and bid quotes of a transaction, $T_{X,P}$ ($T_{X,C}$) is the transaction cost of exercising a put (call), T_S is the cost for trading underlying asset, and T_P (T_C) is the fee for trading put (call) options. These two inequalities show that the value of an American option is no more than the value of a synthetic option plus market frictions. Thus, the degree of violation for the upper boundary of a call option is:

$$VIO`_{CU} = C^b - (P^a + S_0^a - X \cdot e^{-rd,\, a \times \tau}) - (T_{X,P} + T_S + T_P + T_C) \qquad (8)$$

The terms in the first parenthesis of (8) represent the upper boundary of a call without market frictions, and the terms in the second parenthesis of (8) are transaction costs. If $VIO`_{CU}$ is greater than zero, a violation of the upper boundary occurs. The larger the magnitude of $VIO`_{CU}$, the greater the degree of violation tends to be. In the same way, the degree of violation for the upper boundary of an American put option is stated as:

$$VIO`_{PU} = P^b - (C^a - S_0^b \cdot e^{-rf,\, a \times \tau} + X) - (T_{X,c} + T_S + T_C + T_P). \qquad (9)$$

A positive $VIO`_{PU}$ indicates that the upper boundary for put is violated. Thus, the divergence between implied and market prices in the presence of market frictions is expressed as:

$$
\begin{aligned}
DIV` &= VIO`_{CU} - VIO`_{CU} \\
&= C^b - (P^a + S_0{}^a - X \cdot e^{-rd,a \cdot \tau}) - (T_{X,P} + T_S + T_P + T_C) \\
&\qquad - [P^b - (C^a - S_0{}^b \cdot e^{-rf,a \cdot \tau} + X) - (T_{X,C} + T_S + T_C + T_P)] \\
&= (C^a + C^b - P^a - P^b) + (S_0{}^a - S_0{}^b \cdot e^{-rf,a \cdot \tau}) \\
&\qquad + (X - X \cdot e^{-rd,a \cdot \tau}) + (T_{X,C} - T_{X,P}) \qquad (10)
\end{aligned}
$$

II. 2. Discussion of the Option Boundary approach

Section II. 1 describes the option boundary approach to construct the divergence between implied and market prices when market frictions and early exercise effect exist. This approach assumes that implied price is equally likely to lie at any point in the range of possible values derived from option boundaries. That is:

$$Prob.\ (S_0{}^{\wedge} = \alpha_i \mid L \le \alpha_i \le H) = Prob.\ (S^{0\wedge} = \alpha_j \mid L \le \alpha_j \le H),\ for\ \forall\ \alpha_i\ and\ \alpha_j \qquad (11)$$

Prob. is probability. L and H are the lower and higher bounds for the range of implied price, respectively. α_i and α_j are any real number between L and H. If the range is asymmetrically biased to the upside (downside) of the current market price, the implied price is more likely to be greater (less) than the market price. In fact, the bias of the range is measured by the divergences, DIV and DIV`, which is used to capture the information content in option markets.

For example, according to the upper boundaries of call and put options in equations (6) and (7), L and H can be specified as:

$$L = C^b - P^a + X \times e^{-rd,a \times \tau} - (T_{X,P} + T_S + T_P + T_C) \qquad \le S_a{}^0 \qquad (12)$$
$$H = [C^a - P^b + X + (T_{X,C} + T_S + T_P + T_C)] / e^{-rf,a \times \tau} \qquad \ge S_b{}^0 \qquad (13)$$

[L, H] serves as a range estimate for the implied price when boundary conditions are not violated.[2] $VIO`_{CU}$, which equals L minus $S_0{}^a$, measures the difference between the lower bound and the ask side of the underlying asset's price. On the other hand, $VIO`_{PU}$ is measured by $S_0{}^b$ minus H and determines the difference between the bid side of the asset's price and the higher bound. The greater the value of $VIO`_{CU}$ ($VIO`_{PU}$), the more expensive the call (put) premium is

[2] When the boundary conditions hold, the difference between implied and market prices does not exceed $MAX(|L - S_0{}^a|, |S_0{}^b - H|)$. On the other hand, if boundary conditions are violated, the implied price will lie outside the range of [L, H]. However, DIV and DIV` are still the measure for the bias.

relative to the corresponding put (call) premium. Thus, DIV`, which equals $VIO`_{CU} - VIO`_{PU}$ in equation (10), measures the bias of the range estimate for implied price. A positive (negative) DIV` shows that the implied price has a higher probability to be larger (smaller) than the market price. The range and DIV` are graphically illustrated in Figure 1.

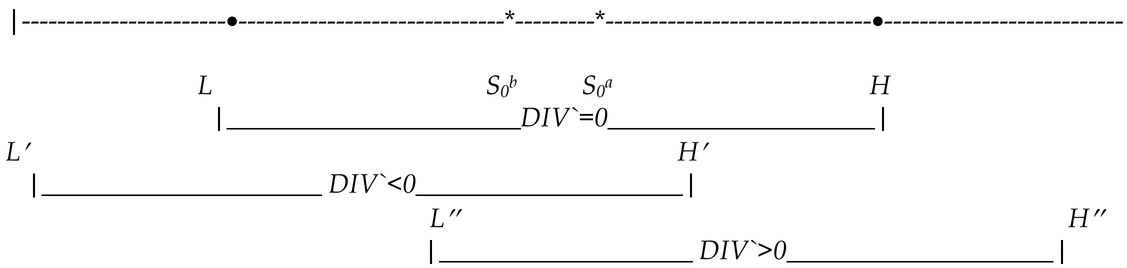

Figure 1
An Illustration for the Calculation of Divergence Between the Implied and Market Prices

In Figure 1, when the range of L to H is not biased to either side, *DIV`* is zero and the underlying asset's price is equally likely to move up or down. However, as the range of L′ (L″) to H′ (H″) is biased to the left (right) side of market price, *DIV`* is less (greater) than zero and the underlying asset's price is more likely to decrease (increase). Thus, *DIV`* measures the bias of the range and reveals the pressure of depreciation or appreciation of an asset's value.

III. Data

To eliminate the measurement error from non-synchronous trading problem, the construction of divergences based on the option boundary approach demands intensive option data that shall include simultaneously recorded trade-by-trade bid, ask, and transaction prices for both underlying assets and options. According to the availability of current option data, the currency option data provided by the PHLX can best satisfy the requirements. The tick-by-tick currency option transaction records complied by the PHLX contain the time of trade (from second to second according to time stamp), expiration code, option premiums (transaction, bid, and ask prices), strike price, and actual spot rates (transaction, bid, and ask rates) reported by Telerate. Because the underlying exchange rates are reported at the exact clock time of each option trade, it is believed that the effect of price non-simultaneity on the estimation of implied parameters is minimal.

For the examination of information content in implied prices, this study also needs the transaction costs of currency options, tick-by-tick exchange rates, and interest rates for risk free claims matching the maturities of the option contracts. The transaction costs are provided by the PHLX and the Options Clearing Corporation (hereafter OCC). Tick-by-tick exchange rate data and daily Eurocurrency market rates are provided by the Olsen Data AG.[3]

The sample period analyzed in this research is from January 1, 1987 to December 31, 1998. There are 1.3 million option transaction records (across all foreign currency contracts traded on the exchange) over this period. Deutsche Mark (hereafter DEM) and Japanese Yen (hereafter JPY) account for 62% of the total transactions. Thus, this paper focuses on these two currencies. A total of 753,317 American options are used in this study.

Among the lines of previous studies (e.g., Tucker (1985), Bodurtha and Courtadon (1986), and El-Mekkaoui and Flood (1998)) using the transaction data provided by the PHLX and OCC, market makers' transaction costs are estimated as follows[4]:

OCC transaction fees (per currency option contract)	
OCC currency option exercise fee	\$1.00
OCC initial fee	\$0.05
Proxy for total exercise costs ($T_{X,C}$ and $T_{X,P}$):	\$1.05
PHLX transaction fees (per currency option contract)	
PHLX currency option transaction fee	\$0.05
PHLX option exchange fee	\$0.07
Proxy for total transaction costs (T_C and T_P)	\$0.12

The option premium spreads, exchange rate spreads, and foreign exchange trading costs are not listed here, since this study employs the actual bid and ask option premium quotes (C^a, C^b, P^a,

[3] The interest rates are available for maturities of 7 days and for, 1, 2, 3, 6 and 12 months. They typically do not change very often for a given trading day and the change is usually small. For those cases in which the date does not match the currency option maturity, linear interpolation is used to estimate the interest rates.

[4] These estimates of the transaction costs are conservative and will vary across different investors and different sample periods. As long as the costs do not fluctuate dramatically over minutes, the test results in this study should not be altered.

and P^b) in the PHLX and the corresponding bid-ask exchange rate quotes (S_0^a and S_0^b) provided by Telerate. Therefore, in constructing the divergences, the estimated market frictions for currency option trading are measured by the bid-ask spreads (for both exchange rates and currency option premiums) plus $1.17 ($1.05+$0.12) per currency option contract.

The exchange rate returns, ($ln(S_{t+1}/S_t)$), are estimated across hourly and daily intervals. Table 1 reports the descriptive statistics for the exchange rate returns. Over the sample period, the data yields approximately 105,168 hourly and 4,382 daily return observations. As indicated by the positive mean values in Table 1, both DEM and JPY slightly appreciated against the U.S. dollar from 1987 through 1998. Also, the data indicates greater fluctuation in JPY exchange rate returns, vis a vis those for DEM. The augmented Dicky-Fuller (ADF) test is performed to examine unit-root problem which would cause spurious regressions. As indicated, all the unit-root hypotheses are rejected at less than 1% level and the foreign exchange return processes are stationary.

Based on the option boundary approach, this study estimates the divergences and boundary violations that are summarized in Table 2. Specifically, an hourly divergence is estimated using a pair of call and put currency options with: (1) the same spot price, (2) the same exercise price, (3) common expiration date, and (4) which are traded within 5 minutes of each other. If there is more than one call (put) matching a specific put (call), the closest traded call (put) is selected for the match. For example, if two put options, traded at 12:30 and 12:31 respectively, match a call option traded at 12:32, then the put traded at 12:31 is selected to match the call. When there is more than one divergence found within an hour, the last (most recent) divergence is used to predict the next hour's exchange rate return. A daily divergence is calculated in the same manner, except that the put and call must be traded within 60 minutes of each other.[5] If currency markets are efficient, at least to the extent that information asymmetry

[5] In practice, as long as interest rates are stable within a day and interest payments are made daily, these pairs are still consistent with the boundary conditions. However, the informational contained in these pairs would be weaker.

across different markets is rapidly eliminated, then the predictability of daily divergences should be weaker than that of hourly divergences. In addition, daily boundary violations are more likely to occur than hourly violations.

Panel 2.A reports divergences and violations for DEM. Over the sample period, there are 3938 divergences. The means of the divergences are not significantly different from zero. For example, the mean for DIV (the difference between the implied and market exchange rates derived from American options with no market frictions) is -1.09 basis points with a standard deviation of 2.27. For DEM daily divergences, the pattern is akin to that of hourly divergences. Panel 2.B reports the divergences and violations for JPY options and suggests a similar pattern to those found in DEM options.

IV. Hypotheses and Empirical Results

IV. 1 Examination of the information content in option markets

If the divergence between implied and market prices is purely attributed to measurement errors and market frictions, it should not contain any information about future price changes, but merely reflect noise. However, if there is information content in option markets, the divergence should be positively related to the future price movements, because the divergence gauges the strength of the deviations between implied and market prices. Similar to previous studies (e.g., Manaster and Rendleman (1982), Tucker (1987), Finucane (1991), Jorion (1995), and Easley, O'Hara, and Srinivas (1998)), the test model for the information content in the divergence is specified as:

$$R_{t+1} = \alpha + \beta_R R_t + \beta_D D_t + \eta_{t+1}, \tag{14}$$

where α, β_R, and β_D are coefficients to be estimated. R_{t+1}, calculated as $ln(S_{t+1}/S_t)$, is return at time t+1, R_t is return at time t, D_t is the divergence (DIV or DIV\`) observed at time t, and η is residual term. R_t in equation (14) is to remove the autocorrelation effect on the test of information content. As such, equation (14) is to examine whether or not the implied price carries information that is not reflected in the current underlying asset's price. If there is information content in the divergence between implied and market prices, β_D should be significantly positive. To allow for potential heteroskedasticity residual properties, the t-statistics provided for (14) and all subsequent regressions are measured using Hansen's (1982) generalized method of moments (GMM) estimation and Parzen weights (Gallant (1987). Since the length of residual autocorrelation in regression is unknown, Andrews' (1991) method of automatic bandwidth selection is employed.

Table 3 reports the information content test results based on equation (14). Panel 3.A shows that the β estimates are 0.000059 and 0.000052 for DIV and DIV`, respectively. Both of them are significant at less than 1% level. It indicates that the hourly divergences for DEM contain the information about the future exchange rate movements. More interestingly, the market frictions seem not to hinder the information content, because both DIV and DIV` are significant. Nevertheless, the test results for daily divergences are insignificant. A possible explanation for the insignificance is that the markets are efficient enough to eliminate information asymmetry across different markets within a short period of time. Panel 3.B presents the test results for JPY and exhibits similar findings.

According to the empirical evidence shown in Table 3, the divergences derived based on option boundary approach do contain the information about the future exchange rate changes. However, the significance of β_D estimates in equation could be driven by the option boundary violations. To investigate the potential impact of boundary violations on the test of the information content, this study removes the boundary violations and retests the equation (14). The model is restated as:

$$R_{t+1} = a + b_R R_t + b_D d_t + \eta_{t+1}, \qquad (15)$$

where d_t is the divergence not associated with any boundary violations. If the information content is purely contributed to the boundary violations, then the estimates of b_D should be insignificant. Table 4 reports the test results on the information content of the divergences after the effect of boundary violations are removed. The findings based on equation (15) are essentially similar to those in Table 3. It suggests that the information content in the divergence is not driven by the boundary violations.

Moreover, as noted in previous studies (Bodurtha and Courtadon (1986), and El-Mekkaoui and Flood (1998)), when an underlying asset's price increases (decreases) dramatically, call (put) option boundaries are more likely to be violated than put (call) boundaries. This study explicitly examines the relation between the boundary violations and market expectations based on the following test model:

$$R_{t+1} = \alpha + \beta_R R_t + \beta_{vio} D_{VIO,t} + \eta_{t+1}, \qquad (16)$$

$$VIO = VIO_{CU},\ VIO_{PU}, VIO`_{CU},\ or\ VIO`_{PU}$$

D_{VIO} refers to the divergence associated with the boundary violations. If option boundary violations convey market expectations about the underlying asset's price changes, the β_{vio} estimates should be significant. Yet, in case these boundary violations are attributed to noise traders who ignore arbitrage force from option boundaries, D_{VIO} should not contain the information about the future price movements. The empirical results on the information content of boundary violations are shown in Table 5. They indicate some support for the predictability of the boundary violations. For example, the β_{vio} of the hourly DEM $VIO`_{PU}$ is estimated at 0.000059 and significant at less than 5% level. It implies that the boundary violations could be attributed to information trading.

IV. 2 Robust Test

A critical problem about the information content tests is the behavior of return process. Foreign exchange return processes could follow a more complicated stochastic process than AR(1). Thus, the significance of the information content tests might not be convincing, because the return processes are not pre-whitened. For a robust examination, we begin by filtering foreign exchange return processes for ARMA-TARCH effects and, subsequently, apply the residuals of the filtering model to equation (14).[6] The general structure of the ARMA(m,n)-TARCH(p,q) model is defined as follows:

$$R_t = a_0 + \sum_{i=1}^{m} \phi_i R_{t-i} + \varepsilon_t - \sum_{j=1}^{n} \theta_i \varepsilon_{t-j} \tag{17}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \text{, where } d_t = 1 \text{ if } \varepsilon_t < 0 \text{ and 0 otherwise.} \tag{18}$$

R_t is foreign exchange return, σ is standard deviation, and ε denotes white noise. The ARMA - TARCH models are estimated by maximum likelihood methods. The *Q*-statistic (Ljung and Box (1979))[7] is employed to examine autocorrelation of the residual process and the AIC and BIC criteria are used to determine the appropriate number of lags. For parsimony reason, this study chooses ARMA(2,1)–TARCH(1,1) for DEM and ARMA(2,2)–TARCH(1,2) for JPY. Table 6 summarizes the selected ARMA - TARCH models which are employed to filter the ARMA–TARCH effects. The Q statistics reported in Table 6 suggest that autocorrelation is not present in any of the series and the residuals computed as a result of fitting ARMA-TARCH models to the returns do not exhibit any remaining ARMA or ARCH effects. For example, the Q-statistic computed up to lag of 10 has a p-value of 94.30% for DEM, which indicates no autocorrelation in the residual process up to lag of 10.

---

[6] The time-series models that have emerged from the original work of Engle (1982) are commonly lumped into a family of models referred to as ARCH. ARCH models are constructed to imbed autoregressive conditional heteroskedastic behavior in the data. Several extensions to Engle's work have been developed, but one set of extensions that have received considerable attention as a result of both their intuition as well as their explanatory power, are the TARCH or threshold autoregressive conditional heteroskedasticy class of models (Glosten, Jagannathan and Runkle (1994)). These models account for the possibility that the market reacts in an asymmetric fashion to good and bad news. Besides TARCH model, we also apply ARCH-M, EGARCH, and various GARCH models. The test results for the information content in the implied prices are qualitatively the same.

[7] The *Q*-statistic is computed in the following manner (Ljung and Box (1979)):

$$Q(L) = T(T-2)\sum_{\ell=1}^{L} \frac{\rho(\ell)^2}{(T-l)}$$

The statistic $\rho(\ell)$ refers to the autocorrelation of the relevant series at lag ℓ. *Q(L)* refers to the *Q*-statistic computed up to a lag L.

Based on the residuals from the ARMA-TARCH models, we re-examine the information content of the divergences. The test model is stated as:

$$RESID_{t+1} = \alpha + \beta_i D_{i,t} + \eta_{t+1}, \qquad (19)$$

RESID stands for the residuals from ARMA-TARCH models. D_t is the divergence (DIV, DIV\`) observed at time t. As indicated in Table 7, the test results for the hourly DIV and DIV\` are still significant, although the t-statistics are lower than before. Thus, the findings in Table 7 confirm the information content of the implied prices derived from the option boundary approach.

V. Conclusions

This paper provides the option boundary approach to examine the information content of implied prices in option markets. Because the previous methodologies of extracting implied prices are insufficient when early exercise effect and model misspecification problem exist, this paper provides the option boundary approach to derive the divergences between implied and market prices.

The divergences are employed to examine the informational content in option markets.

According to the option boundary approach, we construct the divergences for two different cases —American options without market frictions (DIV), and American options with market frictions (DIV\`). Because the option boundary approach requires simultaneously recorded bid, ask, and transaction prices for both underlying asst's prices and options, the PHLX currency options data is employed in this study. The empirical evidence suggests that there is informational content in the implied prices derived from option premiums. However, markets are efficient, at least to the extent that any asymmetric information between the option and spot markets is eliminated within a day. It is also worth noting that option boundary violations are relevant to information trading. Finally, for a robust test, after the ARMA-TARCH effects on the foreign exchange return processes are considered, the test results are still significant.

References

Andrews, D.W.K., 1991, "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometrica*, 59, 817-858.

Bhattacharya, A., 1987, "Price Changes of Related Securities: The Case of Call Options and Stocks, " *Journal of Financial And Quantitative Analysis*, 22, 1-15.

Bodurtha, J. and Courtadon, G., 1986, "Efficiency Tests of Foreign Currency Options Market, "*The Journal of Finance*, 41, 151-162.

Brenner, M., Eom,Y. H., and Landskroner, Y., 1996, "Implied Foreign Exchange Rates Using Option Prices," *International Review of Financial Analysis*, 5, 171-183.

Chakravarty, S., H. Gulen, and S. Mayhew, 2002, "Informed trading in stock and option markets," working paper, 2002

Chan, K., Y. P. Chung, and W. M. Fong, 2002, "The informational role of stock and option volume," *Review of Financial Studies*, 15, 1049-1075.

Chan, K., Y. P. Chung, and H. Johnson, 1993, "why option prices lag stock prices; A trading based explanation," *The Journal of Finance*, 48, 1957-1967.

Cox, J. C. and Rubinstein, M., 1985, *Option Markets*, Prentice-Hall, Englewood Cliffs, New Jersey.

Diltz, J. D., and S. Kim, 1996, "The relationship between stock and option price changes," *Financial Review*, 31, 499-519.

Easley, D., O'Hara, M., and Srinivas, P.S., 1998, "Option Volume and Stock Prices: Evidence on Where Informed Traders Trade," *The Journal of Finance*, 53, 431-465.

El-Mekkaoui, M. and Flood, M. D., 1998, "Put-call Parity Revisited: Intradaily Tests in the Foreign Currency Options Market," *Journal of International Financial Markets*, 8, 357-376.

Engle, R. F. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica,* 50 (1982), 987-1007.

Finucane, T., 1991, "put-Call Parity and Expected Returns," *Journal of Financial And Quantitative Analysis*, 26, 445-457.

Gallant, A.R., 1987, *Nonlinear Statistical Models*, Wiley Press, New York.

Glosten, L. R.; R. Jagannathan; and D. Runkle. "Relationship between the Expected Value and the Volatility of the Nominal Excess Return on Stocks." *The Journal of Finance*, 48 (1994), 1779-1801.

Hansen, L.P., 1982, "Large Sample Properties of Generalized Method of Moment Estimators," *Econometrica*, 50, 1029-1054.

Jorion, P., 1995, "Predicting Volatility in the Foreign Exchange Market," *The Journal of Finance*, 50, 507-527.

Kumar, R., A. Sarin, and K. Shastri, 1992,"The behavior of option price around large block transactions in the underlying security," *The Journal of Finance*, 47, 879-889.

Latané, H. and Rendleman R., 1976, "Standard Deviations of Stock Price Ratios Implied in Option Prices," *The Journal of Finance,* 31, 369-381.

Ljung, G.., and G.. Box. "On A Measure of Lack of Fit in Time Series Models." *Biometrika*, 66 (1979), 265-270.

Malz, A., 1996, "Using Option Prices to Estimate Realignment Probabilities in the European Monetary System: The Case of Sterling-Mark," *Journal of International Money and Finance*, 15, 717-748.

Manaster, S. and Rendleman, R., 1982, "Option Prices as Predictors of Equilibrium Stock Prices," *The Journal of Finance*, 4, 1043 - 1057.

O'Connor, M. L., 1999, "The corss-sectional relationship between trading costs and lead/lag effects in stock & option markets," *Financial Review*, 34, 95-117.

Pan, M. S., Hocking, R. T., and Rim, H. K., 1996, "The Intertemporal Relationship Between the Currency Spot Market and the Currency Option Market," *Journal of Business Finance & Accounting*, 23, 1307-1317.

Stephan, J. and Whaley, R., 1990, "Intraday Price Change and Trading Volume Relations in the Stock and Stock Options Markets," *The Journal of Finance*, 45, 191-220.

Tucker, A.L., 1985, "Empirical Tests of the Efficiency of the Currency Option Market," *The Journal of Financial Research*, 8, 275-285.

Tucker, A., 1987, "Foreign Exchange Option Prices as Predictor of Equilibrium Forward Exchange Rates," *Journal of International Money and Finance*, 6, 283-294.

Table 1
Descriptive Statistics for Exchange Rate Returns

This table reports the observations of the exchange rate return (Obs.), the means of the exchange rate returns (Mean), and the standard deviations (Std.) of the exchange rate changes for Deutsche Mark (DEM) and Japanese Yen (JPY) over the sample period of 01/01/1987 through 12/31/1998. The Augmented Dickey Fuller (ADF) test is used to examine the presence of unit roots and check the stationarity of the return processes. It is conducted from the OLS estimation of equation: $\Delta X_t=\mu+(\rho-1)X_{t-1}+\alpha t+\Delta X_{t-1}+\Delta X_{t-2}+.. +\Delta X_{t-p}+\varepsilon_t$, where X is individual time-series under examination, Δ is the first-order difference operator, μ is the intercept (long-term drift), t is a linear time trend, and ε is a stationary random error (white noise). The null hypothesis is that X is a nonstationary time series and is rejected if $\rho-1<0$. The number of lag, p, is set at 8 in this study. The critical values for the ADF test developed by Mackinnon (1991) are employed to determine statistical significance.

		DEM	JPY
Hourly	Obs.	105,168	105,170
	Mean	0.00014%	0.00032%
	Std.	0.11585%	0.12609%
	Max.	1.74854%	3.29968%
	Min.	-4.85499%	-4.03394%
	ADF	<0.001	<0.001
Daily	Obs.	4,382	4,383
	Mean	0.00325%	0.00761%
	Std.	0.59214%	0.62539%
	Max.	3.54987%	6.22577%
	Min.	-3.66548%	-4.75610%
	ADF	<0.001	<0.001

Table 2
Descriptive Statistics for the Divergences

This table presents the descriptive statistics for the divergences derived from Deutsche Mark (DEM) and Japanese Yen (JPY) over the period from 01/01/1987 through 12/31/1998. The definitions for the divergences are described in Section II.1.

Panel 2. A ~ DEM		
	Hourly divergences and violations	
	DIV	DIV`
Obs.	3938	3938
Mean	-1.09	-0.09
Std.	2.27	1.88
Max.	16.17	17.69
Min.	-19	-12
	Daily divergences and violations	
	DIV	DIV`
Obs.	1795	1795
Mean	-1.00	0.00
Std.	2.20	1.90
Max.	16.17	17.69
Min.	-13	-11
Panel 2. B ~ JPY		
	Hourly divergences and violations	
	DIV	DIV`
Obs.	2471	2471
Mean	-1.61	0.22
Std.	2.85	2.17
Max.	16.08	18.66
Min.	-23	-17
	Daily divergences and violations	
	DIV	DIV`
Obs.	1331	1331
Mean	-1.59	0.31
Std.	2.78	2.08
Max.	16.08	18.22
Min.	-23	-17

Note: The derivation procedures for the divergences are based on the option boundary conditions. The definitions and equation numbers from the text are discussed in Section II.1 and given as follows:

DIV = the divergence for American options without market frictions (5)
DIV`= the divergence for American options with market frictions (10)

Table 3
The Standard Regression Test Results on the Predictability of Divergences

$$R_{t+1} = \alpha + \beta_R R_t + \beta_D D_t + \eta_{t+1}, \qquad (14)$$

This table presents the standard regression test results on the information content of the four different divergences. The definitions for the divergences are described in Section II.1.

Panel 3.A ~ DEM

	Obs.	α	$t(\alpha)$	β_R	$t(\beta_R)$	β_D	$t(\beta_D)$	R^2
	Hourly divergences							
DIV	3938	0.000044	1.28	0.06	3.68***	0.000059	4.28***	0.85%
DIV`	3938	-0.000015	-0.48	0.06	3.78***	0.000052	3.11***	0.63%
	Daily divergences							
DIV	1795	-0.000111	-0.68	-0.01	-0.36	0.000015	0.21	0.01%
DIV`	1795	-0.000125	-0.84	-0.01	-0.33	-0.000020	-0.26	0.01%

Panel 3.B ~ JPY

	Obs.	α	$t(\alpha)$	β_R	$t(\beta_R)$	β_D	$t(\beta_D)$	R^2
	Hourly divergences							
DIV	2471	-0.000011	-0.27	0.03	1.34	0.000037	2.81***	0.40%
DIV`	2471	-0.000078	-2.07***	0.03	1.36	0.000032	1.85*	0.22%
	Daily divergences							
DIV	1331	0.000270	1.30	-0.03	-1.15	0.000033	0.51	0.11%
DIV`	1331	0.000213	1.17	-0.03	-1.10	0.000014	0.16	0.09%

Note: 1. The derivation procedures for the divergences are based on the option boundary conditions. The definitions and equation numbers from the text are discussed in Section II.1 and given as follows:

DIV = the divergence for American options without market frictions (5)

DIV`= the divergence for American options with market frictions (10)

2. * indicates significance at the 10% level.
** indicates significance at the 5% level.
*** indicates significance at the 1% level.

Table 4
The Standard Regression Test Results on the Predictability of Divergences Without Boundary Violations

$$R_{t+1} = a + b_R R_t + b_D d_t + \eta_{t+1}, \qquad (15)$$

This table presents the standard regression test results on the predictability of the four different divergences after removing the observations associated with violations. *D*` is the divergence that is not associated with any boundary violations. All the other terms are the same as those in equation (18). D_1 is not listed because all of the D_1s are violated. The definitions for the divergences are described in Section II.1.

Panel 4.A ~ DEM								
	Hourly divergences without violation							
	Obs.	a	$t(a)$	b_R	$t(b_R)$	b_D	$t(b_D)$	R^2
DIV	3089	0.000051	1.33	0.03	1.86*	0.000043	2.93***	0.40%
DIV`	3191	0.000011	0.33	0.04	2.30**	0.000071	3.81***	0.66%
	Daily divergences without violation							
	Obs.	a	$t(a)$	b_R	$t(b_R)$	b_D	$t(b_D)$	R^2
DIV	1478	0.000098	0.53	0.00	-0.06	0.000092	1.23	0.10%
DIV`	1513	-0.000121	-0.73	-0.01	-0.41	0.000011	0.13	0.01%

Panel 4.B ~ JPY								
	Hourly divergences without violation							
	Obs.	a	$t(a)$	b_R	$t(b_R)$	b_D	$t(b_D)$	R^2
DIV	2077	-0.000004	-0.08	0.00	-0.14	0.000038	2.56***	0.32%
DIV`	2096	-0.000062	-1.58	0.00	-0.13	0.000043	1.99*	0.19%
	Daily divergences without violation							
	Obs.	a	$t(a)$	b_R	$t(b_R)$	b_D	$t(b_D)$	R^2
DIV	1126	0.000233	1.05	-0.03	-1.11	-0.000003	-0.04	0.11%
DIV`	11348	0.000201	1.08	-0.03	-1.08	0.000024	0.23	0.10%

Note: 1. The derivation procedures for the divergences are based on the option boundary conditions. The definitions and equation numbers from the text are discussed in Section II.1 and given as follows:

DIV = the divergence for American options without market frictions (5)
DIV`= the divergence for American options with market frictions (10)

2. * indicates significance at the 10% level.
** indicates significance at the 5% level.
*** indicates significance at the 1% level.

Table 5
The Standard Regression Test Results on the Predictability of Divergences Associated With Boundary Violations

$$R_{t+1} = \alpha + \beta_R R_t + \beta_{vio} D_{VIO_i,t} + \eta_{t+1}, \qquad (16)$$
$$VIO_i = VIO_{CU}, VIO_{PU}, VIO`_{CU}, \text{or } VIO`_{PU},$$

This table presents the test results on the predictability of the divergences associated with boundary violations. VIO refers to the violations of the boundary conditions.

Panel 5.A~DEM

Hourly divergence with violation

	Obs.	α	$t(\alpha)$	β_{Rt}	$t(\beta_{Rt})$	β_{vio}	$t(\beta_{vio})$	R^2
D_{VIOCU}	599	-0.000041	-0.48	0.120373	2.02**	0.000102	1.78*	2.37%
D_{VIOPU}	258	0.000117	0.52	0.087201	1.77*	0.000282	1.12	3.41%
$D_{VIO`CU}$	574	-0.000235	-2.74***	0.077793	0.47	0.000022	1.87*	0.67%
$D_{VIO`PU}$	178	0.000216	1.01	0.130129	0.65	0.000059	2.04**	1.87%

Daily divergence with violation

	Obs.	α	$t(\alpha)$	β_{Rt}	$t(\beta_{Rt})$	β_{vio}	$t(\beta_{vio})$	R^2
D_{VIOCU}	224	-0.000368	-0.89	-0.048574	-1.09	-0.000317	-0.93	1.11%
D_{VIOPU}	100	-0.002102	-2.01***	-0.070066	-0.39	-0.000423	-0.58	0.96%
$D_{VIO`CU}$	223	-0.000425	-1.02	-0.061277	0.03	0.000006	-1.29	0.75%
$D_{VIO`PU}$	62	-0.001157	-1.13	0.143793	-1.44	-0.000663	1.76*	4.16%

Panel 5.B~JPY

Hourly divergence with violation

	Obs.	α	$t(\alpha)$	β_{Rt}	$t(\beta_{Rt})$	β_{vio}	$t(\beta_{vio})$	R^2
D_{VIOCU}	312	-0.000085	-0.61	0.083154	1.28	0.000044	1.87*	1.65%
D_{VIOPU}	87	0.000003	0.01	-0.011555	0.34	0.000065	-0.08	0.18%
$D_{VIO`CU}$	302	-0.000231	-1.73*	0.118208	0.74	0.000025	2.41**	2.10%
$D_{VIO`PU}$	76	0.000056	0.22	-0.103187	0.38	0.000044	-0.89	1.15%

Daily divergence with violation

	Obs.	α	$t(\alpha)$	β_{Rt}	$t(\beta_{Rt})$	β_{vio}	$t(\beta_{vio})$	R^2
D_{VIOCU}	160	-0.000168	-0.23	-0.025564	0.61	0.000113	-0.39	0.28%
D_{VIOPU}	51	0.002015	1.88*	0.036917	-0.37	-0.000366	0.23	0.36%
$D_{VIO`CU}$	159	-0.000142	-0.19	-0.030534	0.18	0.000037	-0.45	0.14%
$D_{VIO`PU}$	42	0.001877	1.47	-0.009884	0.59	0.000324	-0.07	0.71%

Note: 1. The derivation procedures for the boundary violations are discussed in Section II.1 and given as follows:

$$VIO_{CU} = C - S^0 + P - X\, e^{-rd \times \tau} \qquad (3)$$
$$VIO_{PU} = P - X + C - S^0 e^{-rf \times \tau} \qquad (4)$$
$$VIO`_{CU} = C^b - (P^a + S_a^{\,0} - X \times e^{-rd,\, a \times \tau}) + (T_{X,P} + T_S + T_P + T_C) \qquad (8)$$
$$VIO`_{PU} = P^b - (C^a - S_b^{\,0} \times e^{-rf,\, a \times \tau} + X) + (T_{X,c} + T_S + T_C + T_P). \qquad (9)$$

2. "*" indicates significance at the 10% level, "**", 5% level, and "***", 1% level.

Table 6
The ARMA-TARCH models for foreign exchange returns and Ljung-Box Q statistics for Residuals

The ARMA(m,n)-TARCH(p,q) model is defined as follows:

$$R_t = a_0 + \sum_{i=1}^{m} \phi_i R_{t-i} + \varepsilon_t - \sum_{j=1}^{n} \theta_i \varepsilon_{t-j} \quad (17)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \text{, where } d_t = 1 \text{ if } \varepsilon_t < 0 \text{ and 0 otherwise.} \quad (18)$$

The Q-statistic is computed in the following manner (Ljung and Box (1979)):

$$Q(L) = T(T-2)\sum_{\ell=1}^{L} \frac{\rho(\ell)^2}{(T-l)}$$

The statistic $\rho(\ell)$ refers to the autocorrelation of the relevant series at lag ℓ. *Q(L)* refers to the *Q*-statistic computed up to a lag L.

ARMA(2,1)-TARCH(1,1) for DEM				ARMA(2,2)-TARCH(1,2) for JPY			
	ARMA				ARMA		
	Coefficient	Std.	P-value		Coefficient	Std.	P-value
AR(2)	0.0038	0.0146	4.00%	AR(2)	0.6475	0.3779	8.67%
MA(1)	0.0526	0.0166	0.15%	MA(2)	-0.7647	0.3763	4.21%
	Variance Equation				Variance Equation		
	Coefficient	Std.	P-value		Coefficient	Std.	P-value
Constant	0.0000	0.0000	6.18%	Constant	0.0000	0.0000	6.27%
ARCH(1)	0.7353	0.0252	0.00%	ARCH(1)	0.7685	0.0444	0.00%
(RESID<0)*ARCH(1)	-0.0465	0.0377	21.68%	(RESID<0)*ARCH(1)	-0.0282	0.0382	46.04%
GARCH(1)	0.6120	0.0072	0.00%	GARCH(1)	0.4255	0.0372	0.00%
				GARCH(2)	0.1575	0.0244	0.00%
Obs.	105,168			Obs.	105,170		
R-squared	1.77%			R-squared	1.97%		
Akaike info criterion	-11.2240			Akaike info criterion	-11.1896		
Schwarz criterion	-11.2234			Schwarz criterion	-11.1890		
Partial Correlation	Q-Stat	P-value		Partial Correlation	Q-Stat	P-value	
Q(5)	2.0654	55.90%		*Q(5)*	2.1284	71.20%	
Q(10)	2.8501	94.30%		*Q(10)*	2.7120	91.00%	
Q(20)	3.6644	100.00%		*Q(20)*	4.3997	100.00%	

Table 7
The Standard Regression Test Results on the information content of implied prices
(After the removal of ARMA-TARCH effects)

$$RESID_{t+1} = \alpha + \beta_i D_{i,t} + \eta_{t+1}, \qquad (19)$$

This table presents the test results on the information content of the divergences RESID stands for the residuals from ARMA-TARCH models. D_t is the divergence (DIV, DIV`) observed at time t.

Panel 7A ~ DEM hourly divergences						
	Obs.	α	$t(\alpha)$	β_i	$t(\beta_i)$	R^2
DIV	3938	-0.000010	1.28	0.000041	2.44**	0.66%
DIV`	3938	-0.000039	-0.48	0.000019	2.13**	0.39%

Panel 7.B ~ JPY hourly divergences						
	Obs.	α	$t(\alpha)$	β_i	$t(\beta_i)$	R^2
DIV	1795	-0.000011	-0.0000	0.000015	2.17**	0.26%
DIV`	1795	-0.000078	-0.0001	0.000017	1.72*	0.18%

Note: 1. The derivation procedures for the divergences are based on the option boundary conditions. The definitions and equation numbers from the text are discussed in Section II.1 and given as follows:

DIV = the divergence for American options without market frictions (5)

DIV`= the divergence for American options with market frictions (10)

2. * indicates significance at the 10% level.

** indicates significance at the 5% level.