Central Bank Interventions and Jumps in Double Long Memory Models of Daily Exchange Rates*
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Abstract

In this paper, we estimate ARFIMA-FIGARCH models for the major exchange rates (against the US dollar) which have been subject to direct central bank interventions in the last decades. We show that the normality assumption is not adequate due to the occurrence of volatility outliers and its rejection is related to these interventions. Consequently, we rely on a normal mixture distribution that allows for endogenously determined jumps in the process governing the exchange rate dynamics. This distribution performs rather well and is found to be important for the estimation of the persistence of volatility shocks. Introducing a time-varying jump probability associated to central bank interventions, we find that the central bank interventions, conducted in either a coordinated or unilateral way, induce a jump in the process and tend to increase exchange rate volatility.

Keywords: Exchange rate dynamics, ARFIMA process, Fractionally Integrated GARCH, normal mixtures, central bank interventions;
JEL Classification: C22, F41, G15.

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1 Introduction

Given the apparent lack of any structural dynamic economic theory explaining the variations in the first two conditional moments of daily and weekly exchange rates, econometricians have extended traditional time series tools such as Autoregressive Moving Average (ARMA) models for the mean to essentially equivalent models for the variance. The Autoregressive Conditional Heteroscedasticity (ARCH) models (Engle, 1982) and its numerous extensions are now commonly used to describe and forecast changes in volatility of financial time series (see Palm, 1996).

The estimation of these models is usually done by approximate Quasi Maximum Likelihood (QML), assuming that the innovations are normally distributed. Indeed, even if unrealistic, the normality assumption may be justified by the fact that the Gaussian QML estimator is consistent provided the conditional mean and the conditional variance are specified correctly, see Weiss (1986) and Bollerslev and Wooldridge (1992) among others.

The first goal of the paper is to show, given the specification choice of the first two conditional moments, that the occurrence of outliers is primarily responsible for the rejection of the Gaussian assumption. In turn, these outliers may be caused by specific financial events like direct central bank interventions in the foreign exchange market. Therefore, the occurrence of these events have strong implications for the modelling strategies regarding these series. Accordingly, in order to model this feature, we introduce a normal mixture distribution, the Bernoulli-normal that allows for the possibility of endogenously determined jumps. We find that for three out of the four considered exchange rates a mixture distribution turns out to be supported by the data.

While capturing the short run dynamics of exchange rates, the constant jump probability specification yields few economic and financial insights. Building on the empirical literature on direct central bank interventions in the foreign exchange markets (Dominguez, 1998), we extend the basic normal mixture model and introduce a time-varying jump probability which is associated to the direct purchases and sales of foreign currency conducted by the major central banks. It is found that the central bank interventions, carried out either in a coordinated or in an unilateral way, induce a jump in the process and thus tend to increase exchange rate volatility.

The paper is organized as follows. Section 2 describes the specification choice retained for the conditional mean and the conditional variance, presents several test statistics and describes the dataset used in the empirical application. Section 3 is devoted to the outliers detection issue and presents the Bernoulli-normal distribution. Section 4 extends the previous analysis by modelling the jump probability while Section 5 concludes.

2 Double Long Memory Models

Over the last decade, the analysis of high frequency financial time series data has focused on the long memory property. As an example, weekly and daily exchange rate returns have been found to be well characterized by fractionally integrated processes. Until recently, the empirical studies have been concerned with fractional roots in either the conditional mean or in the conditional variance of these returns.

2.1 ARFIMA-FIGARCH

In the conditional mean, the discrete time series representation of a fractionally integrated process has been introduced by Granger and Joyeux (1980) and Hosking (1981). Denoting L as the lag operator ($L^k y_t = y_{t-k}$, with $k \geq 0$), and by replacing the difference operator $(1-L)$ of an ARIMA process with the fractional difference operator $(1-L)^\zeta$, where ζ captures the degree of fractional integration, they define the so-called Autoregressive Fractionally Integrated Moving Average (ARFIMA) process. Noting y_t as one hundred times the return of exchange rate s_t ($y_t = 100 * [\ln(s_t) - \ln(s_{t-1})]$), where s_t is the price of the currency under investigation in terms of the USD, the ARFIMA (n, ζ, s) model is formally defined as:

$$\Psi(L)(1-L)^\zeta (y_t - \mu) = \Theta(L)\varepsilon_t \quad (1)$$

$$\varepsilon_t = \sigma_t z_t, \quad (2)$$

where μ is the unconditional mean of process (1), $\Psi(L) = 1 - \psi_1 L - ... - \psi_n L^n$ and $\Theta(L) = 1 + \theta_1 L + ... + \theta_s L^s$ are the usual AR and MA lag polynomials of respective orders n and s (with all roots lying outside the unit circle), σ_t is a time-varying conditional variance (see below) and z_t is an independent and identically distributed (*i.i.d.*) random variable with zero mean. For ease of notation, let us rewrite y_t as $y_t = \bar{\mu}_t(\mu, \Psi, \Theta, \zeta) + \varepsilon_t$, where $\bar{\mu}_t = E(y_t|\Omega_{t-1})$, i.e. the conditional mean of y_t, Ω_t is the information set at time t, $\Psi = (\psi_1, ..., \psi_n)'$ and $\Theta = (\theta_1, ..., \theta_s)'$.

Obviously, $\zeta = 0$ corresponds to a stationary process in the exchange rate returns. If ζ lies between 0 and 1/2, the process is stationary and is said to be persistent. If ζ lies between 0 and -1/2, the process displays some short memory and is said to be antipersistent.[1] The major exchange rates (except the British pound) have been found to display long memory properties by Cheung (1993) and thus their dynamics may be expected to be matched rather well by the ARFIMA specification.

Similar research on the volatility side (Baillie et al., 1996a) has led to an extension of the ARFIMA representation in ε_t^2, leading to the Fractionally Integrated GARCH (FIGARCH) model. The FIGARCH (p, d, q) process is given by:

$$\sigma_t^2(\omega, \beta, \phi, d) = \omega + \left\{1 - [1-\beta(L)]^{-1}\phi(L)(1-L)^d\right\}\varepsilon_t^2, \quad (3)$$

where σ_t^2 is the conditional variance of y_t, d is the fractional degree of integration of ε_t^2; $\beta = (\beta_1, ..., \beta_p)', \phi = (\phi_1, ..., \phi_q)', \beta(L) = \beta_1 L + ... + \beta_p L^p$ and $\phi(L) = 1 - \phi_1 L - ... - \phi_q L^q$ are the lag polynomials of respective orders p and q of which all roots lie outside the unit circle. An interesting feature of the FIGARCH model is that it nests both the GARCH (Bollerslev, 1986) model for $d = 0$ and the IGARCH (Engle and Bollerslev, 1986) model for $d = 1$. As advocated

[1] Furthermore, if $\zeta < -1/2$, the process is non invertible. If $\zeta > 1/2$, the process is not stationary while if $\zeta = 1$, the process has a unit root.

by Baillie et al. (1996a), the IGARCH process may be seen as too restrictive as it implies the infinite persistence of a volatility shock. Such dynamics are contrary to the observed behavior of agents and do not match the persistence observed after important events (see Baillie et al., 1996a; Bollerslev and Engle, 1993). By contrast, for $0 < d < 1$, the FIGARCH model implies a long memory behavior, i.e. a slow decay of the impact of a volatility shock. The FIGARCH class of processes is not covariance stationary, but is strictly stationary and ergodic for $d \in [0, 1]$. For a FIGARCH $(1, d, 1)$ model, sufficient conditions for the conditional variance to be strictly positive are given in Baillie et al. (1996a).[2]

The estimation of the ARFIMA-FIGARCH model is done by approximate (Q)ML.[3] Following the standard procedure in the literature, the truncation order of the infinite polynomials $(1 - L)^{\zeta}$ and $(1 - L)^{d}$ is set to 1000 lags while initial conditions have been set to $\varepsilon_{t^*} = 0$ and $\varepsilon_{t^*}^2 = E\left(\varepsilon_t^2\right)$ for $t^* = 0, -1, -2, ..., -1000$ and $t = 1, 2, ..., T$, where T is the number of observations.[4] Note that Teyssière (1997) shows that the approximate (Q)ML estimates have nice properties: root-n consistency, asymptotic normality and negligible bias.

Applications of FIGARCH models to exchange rates were first proposed by Baillie et al. (1996a) for the DEM and by Tse (1998) for the YEN. Both papers estimate a FIGARCH $(1, d, 0)$ model but do not consider the case of a fractional root in the mean. The results of Baillie et al. (1996a) suggest that the FIGARCH model is much closer to the IGARCH model (but nevertheless different in its implications) while Tse (1998) shows that it exhibits a stable GARCH type of behavior.[5]

Similarly, estimation of ARFIMA processes with time-varying heteroskedasticity is fairly new in the literature. Baillie et al. (1996b) estimate an ARFIMA (n, ζ, s)-GARCH (p, q) process for the post-war inflation rates of several industrial countries while Tschernig (1995) and Lecourt (2000) have found evidence of long memory in the conditional mean of exchange rate returns computed on a high frequency basis. The joint estimation of fractional processes both in the mean and in the variance has been recently proposed by Teyssière (1997). Such a model is referred to as a double long memory or ARFIMA-FIGARCH model. Recent applications to high frequency exchange rate returns have been proposed by Teyssière (1998) and by Beine, Laurent and Lecourt (2002). The results suggest that double long memory models may characterize the dynamics of exchange rate returns.

2.2 Tests

A critical part of our analysis will be devoted to diagnostic tests, either to discriminate between competing distributions or to assess the relevance of the ARFIMA-FIGARCH framework. We will

[2]Some of these sufficient conditions are overly restrictive. For instance, they specify $\omega > 0$. By contrast, our estimation procedure allows ω to be negative but, following Nelson and Cao (1992), checks the positiveness of the conditional variance on a case-by-case basis.

[3]The estimations have been carried out using Gauss 3.6. The results have been reproduced using a slightly modified version of G@RCH 2.2 (see Laurent and Peters, 2002), an Ox package with a friendly dialog-oriented interface dedicated to the estimation and forecast of various univariate ARCH-type models.

[4]Furthermore, it is possible to use presample values in order to estimate the ARFIMA part, as proposed by Teyssière (1997). The change in the results is marginal, so that we use the whole sample to estimate the model.

[5]For an explanation of these divergent results, see Beine, Laurent and Lecourt (2002).

be primarily basing our assessments on five different evaluation procedures.

The first two tests are conducted on the standardized residuals denoted by z_t. The first one concerns estimators of the skewness and excess kurtosis coefficients b_3 and b_4. The second one is the test statistic proposed by Brock et al. (1987), denoted BDS (m), where m is the embedding dimension.[6] The BDS test checks the null hypothesis of *i.i.d.* residuals. This hypothesis is important because our other two evaluation procedures, a nonparametric rank test and the Pearson goodness of fit test, require independent observations. Thus, by failing to accept the *i.i.d.* hypothesis in the standardized residuals, it would be unclear how to correctly interpret a rejection of the null hypothesis implied by these two tests.

The third is a nonparametric rank test introduced by Wright (1998). This test can be used as a misspecification test suitable for GARCH and FIGARCH models. For fixed l, the test statistic $S(l)$ is given by:

$$S(l) = T \sum_{i=1}^{l} \rho(s_{1t}, s_{1t-i})^2, \tag{4}$$

where $\rho(.,.)$ denotes the sample autocorrelation function and s_{1t} is given by:

$$s_{1t} = \left[r(z_t^2) - \frac{T+1}{2} \right] \bigg/ \sqrt{\frac{(T-1)(T+1)}{12}}\ , \tag{5}$$

where $r(z_t)$ is the rank of z_t among $z_1, z_{2,...}, z_T$. Under the null of a correct specification in the conditional variance, Wright (1998) proposes to use a $\chi^2(l)$ distribution. The motivation for using this test is that it is more powerful than alternative tests when z_t is highly non-normal, which seems particularly relevant in financial data.

Finally, since our objective is to assess the relevance of various underlying distributions, our last test is the Pearson goodness-of-fit test that compares the empirical distribution of z_t with the theoretical one. In order to carry out this testing procedure, it is necessary to first classify the residuals in cells according to their magnitude, see Palm and Vlaar (1997) for more details. For a given number of cells denoted g, one computes the following test statistic:

$$P(g) = \sum_{i=1}^{g} \frac{(n_i - En_i)^2}{En_i}, \tag{6}$$

where n_i is the number of observations in cell i (based on the ML estimation) and En_i is the expected number of observations (based on the ML estimates). For *i.i.d.* observations, under the null of a correct distribution, $P(g)$ is distributed as a $\chi^2(g-1)$.[7] As explained by Palm and Vlaar (1997), the choice of g is far from being obvious. For $T = 2252$, these authors set g equal to 50. Given that the number of cells must increase at a rate equal to $T^{0.4}$, we use $g = 70$ for our full sample size (Kendall and Stuart, 1967).

[6] More precisely, we report the t-statistics of these measures. The distance measure is chosen according to the spread of the data (see Brock et al., 1987). For all series, we end up with a distance measure equal to 0.6. Notice that the conclusions of the tests are robust across all possible values for m.

[7] Actually, the asymptotic distribution of $P(g)$ is bounded between a $\chi^2(g-1)$ and a $\chi^2(g-k-1)$ where k is the number of parameters. Since our conclusions hold for both critical values, we report the significance levels relative to the first one.

We also provide the Schwarz (SC) information criterion to compare the different specifications. This statistic complements the evaluation provided by likelihood ratio tests when the distributions are nested.

2.3 Data Description

The analysis has been carried out on the four major currencies against the US dollar (USD), i.e. the Deutsche mark (DEM), the Japanese yen (YEN), the French franc (FRF) and the British pound (GBP) using daily data over the period 1980-1996 (the number of observations T equals 4221 for the YEN and 4313 for the other currencies). The data are provided by the International Bank for Settlements and refer to the spot market. These exchange rates data are mid rates quoted respectively in Frankfort at 2:00 pm (GBP, FRF and DEM) and in Tokyo (YEN) at 10:00 am each day.

3 Mixture Distributions

The choice of an appropriate distribution in the ML estimation procedure is an important issue. The normality assumption may be justified by the fact that the Gaussian QML estimator is consistent provided the conditional mean and the conditional variance are specified correctly.

3.1 Non-normality and outliers detection

Preliminary results based on the Gaussian QML - not reported here to save space, but available upon request - indicate that for the four currencies, we fail to reject at conventional significance levels the null of no fractional integration in the mean. Therefore, we reestimate the models constraining ζ to be zero. This questions the relevance of long memory behavior in exchange rate returns and contrasts with the results of Cheung (1993), Tschernig (1995) and Lecourt (2000). The estimates of the fractional degree of integration in the variance (d) are in line with the findings of Tse (1998), Teyssière (1998) or Beine, Laurent and Lecourt (2002): d equals 0.58, 0.64, 0.39 and 0.30 respectively for the DEM, FRF, GBP and the YEN. Misspecification tests based on the nonparametric rank tests appear to validate the ARMA-FIGARCH framework and the BDS tests support the *i.i.d.* hypothesis for the standardized residuals. As a whole the ARMA-FIGARCH framework seems to match the dynamics of daily exchange rate returns and is a satisfying starting point to study the nature of the underlying distributions.

Turning our attention to the relevance of the distributions, most of the results, unsurprisingly, question the relevance of the Gaussian assumption. The use of the normal distribution leads to excess skewness and excess kurtosis. As a consequence, this density is clearly rejected by the Pearson goodness-of-fit tests.

The excess skewness and kurtosis may be due to the occurrence of numerous financial and monetary events that have taken place during our period of investigation. These events might lead to the occurrence of “Level and Volatility Outliers” (see Hotta and Tsay, 1998) that the normal distribution cannot take into account. A usual approach to identify outliers is the mean average deviation (MAD) procedure. In this procedure, an observation is characterized as an outlier if $y_t > |\kappa[c * med(|y_t - med(y_t)|]|$, where med is the median operator. c is a constant and is derived from $(1/q_{0.75})$ where $q_{0.75}$ is the 75th percentile of the normal distribution. The choice of κ remains arbitrary but values of 2 or 3 are commonly used in practice. Applying the MAD procedure leads to either 91 (if $\kappa = 3$) or 337 (if $\kappa = 2$) outliers for the DEM. For the other exchange rates the respective figures are 56 and 100 for the FRF, 122 and 422 for the GBP and 118 and 377 for the YEN.

Alternatively, Franses and Ghijsels (1999) propose an interesting approach to identify "additive outliers" (AO) in the volatility. Although Franses and Ghijsels (1999) apply this procedure to a GARCH $(1,1)$ model, the extension to a FIGARCH model is straightforward. The procedure is carried out in a sequential way and requires five steps, see Franses and Ghijsels (1999) for details. Applying the Franses and Ghijsels' approach to our data allows us to quantify the number of "aberrant observations", to identify these outliers and to yield AO corrected returns. The procedure leads to the identification of 103 outliers for the DEM, 105 for the FRF, 100 for the GBP and 127 for the YEN. The results suggest that the presence of outliers is primarily responsible for the rejection of the normality assumption: adjusting for these outliers and reestimating the model leads to a dramatic decrease of excess skewness and excess kurtosis on the standardized residuals (complete results are also available upon request). The normality of the AO adjusted returns is supported for the DEM and to some extent for the FRF. The rejection levels for the GBP and the YEN have also significantly decreased. It is worth pointing out that the estimated persistence of volatility shocks (d) is significantly reduced (except for the YEN) and is much more similar across currencies (about 0.36).

The high number of identified outliers for all the investigated currencies calls for the use of another model. One way to reconsider the model is to introduce an endogenous jump through the use of a normal mixture distribution. This jump probability may be related to financial variables thought to influence the dynamics of short run flexible exchange rates.

3.2 Bernoulli-normal

In order to account for the numerous outliers detected in the previous subsection, we rely on a jump-diffusion ARCH-type model that assumes that the returns are drawn from a mixture of normal distributions, i.e. a diffusion process combined with an additive jump process.[8] Let us define the following mixture process as follows:

$$y_t = \bar{\mu}_t + \sigma_t z_t, \text{ with probability } (1-\lambda) \quad (7)$$

$$y_t = \bar{\mu}_t + \sigma_t z_t + \tau + \sqrt{\delta^2} z_t^*, \text{ with probability } \lambda, \quad (8)$$

[8]The following specification is similar to the one proposed by Neely (1999). This author considers this framework in the context of a Bernoulli-Student distribution.

where z_t and z_t^* are *i.i.d.* $N(0,1)$, $E(z_t\ z_t^*) = 0$, λ stands for the probability of a jump and is drawn from a Bernoulli distribution $(0 < \lambda < 1)$, τ is the mean of the jump distribution while δ^2 captures the variance of the jump distribution. $\bar{\mu}_t = \bar{\mu}_t(\mu, \Psi, \Theta, \zeta)$ and $\sigma_t^2 = \sigma_t^2(\omega, \beta, \phi, d)$ are respectively the conditional mean and conditional variance of the diffusion process and are defined as in Section 2.1.

This model can be rewritten as:[9]

$$\begin{aligned} y_t &= E(y_t|\Omega_{t-1}) + \varepsilon_t \qquad (9) \\ \varepsilon_t &\sim (1-\lambda)N(-\lambda\tau, \sigma_t^2) + \lambda N(\tau - \lambda\tau, \sigma_t^2 + \delta^2), \qquad (10) \end{aligned}$$

where $E(y_t|\Omega_{t-1}) = \bar{\mu}_t + \lambda\tau$, where $\lambda\tau$ is the conditional mean of the jump process. Notice that in this specification, λ is assumed to be constant over time.

The log-likelihood associated with this distribution takes the following form:

$$L_{Bern} = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left\{ \frac{(1-\lambda)}{\sigma_t^2} \exp\left[-\frac{(y_t - \bar{\mu}_t)^2}{2\sigma_t^2}\right] + \frac{\lambda}{\sqrt{\sigma_t^2 + \delta^2}} \exp\left[-\frac{(y_t - \bar{\mu}_t - \tau)^2}{2(\sigma_t^2 + \delta^2)}\right]\right\}. \qquad (11)$$

It can be seen that δ^2 is the additional volatility related to the jump. It should be stressed that while the normal mixture distribution can account for excess skewness, its introduction will also affect the conditional fourth moment of the residuals of our ARFIMA-FIGARCH model (see on this point Appendix B of Vlaar, 1994).

One problem with mixture distributions is that the Pearson goodness-of-fit test presented in Section 2 is no longer valid. The reason lies in the fact that, unlike for the normal and Student-t distributions, standardization will not lead to *i.i.d.* residuals in a model with time dependent variance. Palm and Vlaar (1997) have attempted to solve this problem by redefining the sorting mechanism of the residuals. We thus use normalized residuals (z_t^n) instead of standardized residuals (residuals divided by their standard deviation). Normalized residuals are obtained by reexpressing Eq. (10) to have $N(0,1)$ innovations (if the mixture of normal assumption holds), i.e. $z_t^n = F^{-1}\left[(1-\lambda)F\left(\frac{y_t - \bar{\mu}_t}{\sigma_t}\right) + \lambda F\left(\frac{y_t - \bar{\mu}_t - \tau}{\sigma_t + \delta}\right)\right]$, where $F(.)$ and $F^{-1}[.]$ are respectively the cumulative distribution function and the quantile function of the standard normal density.

Note that lag order selection issues are important when building a dynamic model. To determine the orders n, s, p and q of the ARFIMA (n, ζ, s)-FIGARCH (p, d, q) we rely on the Schwarz Bayesian Information Criterion which is known to lead to a parsimonious specification.[10] In a first step, we select the AR and MA terms, assuming a FIGARCH $(1,1)$ specification. Then, given the obtained ARMA specification (choice of n^* and s^*), we compute the information criterion in order to choose the FIGARCH orders.[11] As shown in Table 1, an AR (1) specification has been

[9] Vlaar and Palm (1993) show that under this mixture of normal distributions, $E(\varepsilon_t) = 0$. This is done by shifting the density by $\lambda\tau$. See these authors for more details.

[10] The use of such an information criterion in ARFIMA-FIGARCH models remains to be investigated. Such an investigation, while interesting in its own right, is beyond the scope of this paper.

[11] Note that one could reiterate the selection procedure for the ARMA specification based on the optimal FIGARCH parameterization. After one additional iteration, we obtain the same lag orders n^* and s^* as the ones reported in Table 1.

retained for the DEM, FRF and the YEN while the GBP does not require to include ARMA components in the conditional mean. Concerning the conditional variance, a FIGARCH $(1, d, 1)$ has been retained for the DEM and a FIGARCH $(2, d, 0)$ for the other currencies.

[INSERT TABLES 1 AND 2 HERE]

Table 2 reports the estimation results for the Bernoulli-normal distribution. As a whole, the results confirm the relevance of the ARMA-FIGARCH framework in capturing the dynamics of exchange rates. All the rank tests fail to reject the null hypothesis of an appropriate model. Interestingly, the null of a fractionally integrated process in the conditional mean is still rejected for all currencies.

It may be seen that the contribution of the normal mixtures lies in the important decrease in the excess skewness of the residuals. For the YEN and the DEM, which were previously found to exhibit a skewed distribution, the skewness of the normalized residuals is close to zero and statistically insignificant. This is also true for the FRF and the GBP. For all the currencies under investigation, LR tests confirm that the mixture distribution outperforms the normal distribution regardless the individual significance for each of the three additional parameters (λ, τ and δ^2). Hence, it seems relevant over such a long period to introduce the possibility of breaks and jumps in the dynamics of exchange rate returns. Turning to the values of the parameter estimates, we observe that for the DEM and the FRF, the size of the jump τ is insignificant while the additional volatility associated with the jump (δ^2) is significant. For the YEN, τ is significantly negative, which means that on average, the jump also shifts the density to the left. This implies that the estimated normal mixture model captures mostly the volatility outliers.

Goodness-of-fit tests indicate that the Bernoulli-normal distribution is appropriate for capturing the dynamics of the DEM-USD exchange rate. Regardless the value of g, i.e. the number of cells used in the testing procedure, the relevance of this distribution is supported at conventional significance levels. Unsurprisingly, the same conclusion applies for the FRF (at a 5% nominal level), which is tightly linked to the DEM over the whole sample period through the European Exchange Rate (ERM) mechanism. In the case of the GBP, the evidence in favor of the mixture distribution is not as strong but the relevance of the Bernoulli-normal is still supported by the Pearson test. Finally, the results for the YEN emphasize the need for either another distribution to capture the dynamics of exchange rate returns or the introduction of explicit explanatory variables in the conditional mean. Nevertheless, it should be emphasized that the individual significance of each parameter specific to the mixtures shows the relevance of introducing jumps in the process. It is also worth noting that the rejection level of the $P(g)$ test is much less severe than for the normal distribution. Interestingly, these results are in line with our preliminary procedure adjusting the data for the detected outliers: the filtered data for the DEM, the FRF and the GBP also lead to the acceptance of a conditional normal distribution.

One problem of the mixture distribution lies in its failure to specifically account for excess kurtosis. Relying on the normalized residuals, almost all of the statistics for excess kurtosis b_4 are found to be significant at the 5% level. Nevertheless, the b_4's turn out to be much lower than those obtained for the normal distribution, which confirms that accounting for a non uniform flow of information reduces excess kurtosis. The occurrence of excess kurtosis in general and the rejection of the relevance of the mixture distributions for the YEN in particular, suggests the need for a further investigation of alternative distributions. Such an analysis will be carried on in the next section, in which we model a time-varying jump probability.

In general, the use of different distributions led to relatively similar estimates. In particular, these results confirm the rejection of fractional differencing in the conditional mean. The parameter estimates are found to be quite similar across these distributions and the same model is selected. However, it may be seen that for the estimates of d, i.e. the degree of fractional integration in the variance, the values obtained with the normal mixture are lower than those found with the normal one. This is understandable given that jumps, which otherwise may be spuriously associated with additional volatility, are fully accounted for in the mixture distribution. A related reason is that the mixture specifies an additional parameter δ^2, i.e. the volatility associated with the jumps. The decrease of the persistence of shocks when accounting for jumps in the exchange rate dynamics is emphasized in a recent literature focusing on accounting for long memory or on modelling structural changes as substitutes for each other (Diebold and Inoue, 1999; Granger and Hyung, 1999). The decrease in the estimate of d obtained here with daily data and static structural changes agrees with those of Boubel and Laurent (2001) with intradaily data and Beine and Laurent (2001) with dynamic structural changes (through a Markov Switching process). The results concerning the value of d are also in line with the previous findings of Tse (1998) and Beine, Laurent and Lecourt (2002) who find moderate long run dependence in the volatility of daily exchange rates.

4 Time-Varying jump probability

Central bank interventions are thought to play an important role in explaining the dynamics of exchange rates. It is for instance well known that the day after the Plaza agreement (September 1985) was made public, the dollar depreciated by 4%. It is nevertheless unclear whether the central bank interventions themselves or the expectations built by the market caused these large depreciations. A growing literature has recently investigated the empirical consequences and effectiveness of the major central bank interventions, using either GARCH estimates of the conditional volatility (Baillie and Osterberg, 1997a and b and Dominguez, 1998 among others) or relying on the implied volatilities from the currency option prices (Bonser-Neal and Tanner, 1996). As emphasized in these papers, the effects are best captured with daily exchange rates since interventions are made on a daily basis.

Baillie and Osterberg (1997a and b) find that the conditional mean of exchange rate returns (in the spot or in the forward market) has been mildly affected by some of the interventions conducted by the Fed, the Bundesbank and the Bank of Japan. As shown by Baillie and Osterberg (1997b) and Beine, Bénassy and Lecourt (2002), this effect stands in sharp contrast with the objectives of the Plaza Agreement (i.e. depreciating the dollar) and is explained with the "leaning against the wind" phenomena. As reported by Bonser-Neal and Tanner (1996), the evidence is mixed when using official data of central bank interventions. More precisely, the presence of significant effects depends upon (*i*) the investigated currencies, (*ii*) the sub-period under consideration[12] and (*iii*) the specification in the GARCH process.[13]

The use of central bank interventions as explanatory variables raises several important statistical problems. The main drawback of this approach is that (*i*) it fails to validate the underlying distribution, even after the introduction of the control variables and (*ii*) it fails to investigate the extent to which these control variables explain the empirical distribution. As an alternative to the inclusion of intervention data in a conditional volatility framework, one can rely on an underlying distribution that specifically accounts for jumps in the dynamics of exchange rate returns. An example of this type of distribution is a normal mixture that endogenizes the probability of a jump in the process. Analyzing the behavior of EMS exchange rate dynamics, Vlaar and Palm (1993) show that these mixtures can reduce the skewness. Similar results are obtained by Neely (1999). Palm and Vlaar (1997) also document the relevance of this distribution for the major flexible exchange rates in a GARCH framework.

As shown in Table 2, the Bernoulli-normal distribution fails to be validated by goodness-of-fit tests for the YEN exchange rate over the 1980-1996 period. Furthermore, the empirical distributions still exhibit some excess kurtosis. Consequently, these results emphasize the need for another specification. We build on the normal mixture model and extend the framework by allowing the jump probabilities to vary over time.

Before turning to the introduction of these time-varying jump probabilities, it is important to compare our detected outliers with the estimated number of jumps. Using the MAD procedure with $\kappa = 2$, we detected 337 outliers on the DEM over the full period, which is comparable to the average number of jumps (280) implied by our model. Since we intend to associate jumps with central bank interventions, it is also interesting to compare the estimated number of jumps and the intervention data. The official intervention data on the DEM/USD exchange rate market of the Federal Reserve and the Bundesbank indicate an occurrence of respectively 215 and 264 daily interventions over the 1985-1995 period (the longest available period).[14] Out of these interventions, 97 are coordinated, i.e. they occur the same day (and in the same direction). This leads to 441 days of interventions in this particular market. Our Bernoulli-normal model suggests a total of about 280 jumps for the DEM/USD rate, which seems reasonable if one admits that only a subset of interventions actually causes a move in exchange rate returns.

[12]This is obvious for instance from the results of Dominguez (1998).

[13]As shown by Dominguez (1998), the results may be different depending on whether one uses reported rather than actual interventions and depending on whether one relies on net amounts or only on dummy variables that simply capture the presence of central banks in the market.

[14]Source: the Federal Reserve and the Bundesbank. For an overview on the data and on the intervention policies, see Dominguez (1998). The Bank of Japan interventions data are not available before 1991. As an alternative, it is possible to use interventions reported in the press but one cannot of course exclude reporting errors.

Time-varying jump probabilities have been extensively used in the analysis of exchange rates in the context of ERM currencies (see among others Nieuwland et al., 1994; Neely, 1999 or Vlaar, 1994). Indeed, in this case, the jump probability may be associated to a realignment probability, although it is shown that it can capture other financial events. In this context, unemployment, inflation and/or interest rates differentials (with respect to Germany) are natural candidates as explanatory variables of the jump probability. In the context of flexible exchange rates, the choice of explanatory variables is less straightforward but our starting hypothesis relates jumps to central bank interventions. Thus, in Eq. (7)-(11), we replace λ by λ_t, with λ_t given by the following logit specification:

$$\lambda_t = 1 - (1 + \exp(\gamma_0 + \sum_{i=1}^{M} \gamma_i x_{i,t})^{-1}, \tag{12}$$

where the $x_{i,t}$ are the central bank intervention variables expected to be related to the jump probability.[15] This analysis is carried out for the DEM and the YEN (as daily central bank interventions data are not available for the Bank of France and the Bank of England) for the 1985-1995 period (the longest available period).

Following Dominguez (1998), we separate individual interventions (i.e. interventions conducted on an unilateral basis by a single central bank) from concerted ones (i.e. interventions occurring simultaneously and in the same direction[16]) as the latter may exert a different impact. All variables are dummy (0-1) variables and are exclusive (i.e. the other intervention variables take a value of zero when a particular variable is equal to 1), which allows us to compute the additional jump probability explained by each variable. As a first specification, we introduce three variables: $x_{1,t}$ is a dummy variable that takes 1 if and only if the Federal Reserve intervenes at time t on an individual basis[17] (regardless the direction of the intervention), 0 otherwise; these interventions amount to 65 in the DEM market and to 113 for the YEN; $x_{2,t}$ is a dummy variable that takes 1 if and only if the Bundesbank (167 official interventions) or the BOJ (69 reported interventions)[18] intervenes at time t on an individual basis (regardless the direction of the intervention), 0 otherwise; $x_{3,t}$ is a dummy variable that takes 1 if and only if both central banks intervene at time t (97 occurrences), 0 otherwise.

Note that since the sample size has changed, we have repeated the two-steps model selection procedure explained in the previous section. As shown in Table 3, we end up choosing an ARMA

[15] The definition of $x_{i,t}$ depends on the quotation of the exchange rate. The hours of interventions are not available but some insights have been given by Dominguez (1999) on the basis of Reuters stamps. For the YEN, since the quotation of the exchange rate refers to opening prices, $x_{i,t}$ will capture central bank interventions of both banks the day before the quotation, i.e. at time $t-1$. For the DEM, the lag procedure is less straightforward. Interventions are conducted either before 2.00 pm or after 2.00 pm, depending on whether these are coordinated or not. Most coordinated interventions take place after 2.00 pm to take advantage of the time overlap between the US and the German markets. This is not required for unilateral interventions. Therefore, we consider unilateral operations of the Bundesbank at time t while using coordinated and Fed unilateral interventions at time $t-1$.

[16] A careful inspection of the data reveals that the two involved central banks never intervene in opposite directions.

[17] Since daily exchange rate data refer to 2:00 pm quotations on the London market, we take the one-day lagged official intervention of the Federal reserve to account for time discrepancies.

[18] Since the BOJ does not make their official data available before April 1991, we use interventions reported by the press as a proxy to official ones. The reported interventions data are taken from the Wall Street Journal over the 1985-1991 period (we are grateful to C. Bonser-Neal for kindly transferring the data), and from the Financial Times over the 1992-1995 period.

$(0,0)$-FIGARCH $(1,d,0)$ model for the YEN while we select an ARMA $(1,0)$-FIGARCH $(2,d,0)$ for the DEM. As before, we drop the ζ parameter, i.e. the long memory in the conditional mean, which is found to be insignificant in both cases.

[INSERT TABLE 3 HERE]

Tables 4 and 5 report the estimation results. For the sake of comparison, the first column reports the results for the constant jump probability. The column denoted "Interventions I" corresponds to the time-varying probabilities. Note that the values of the jump parameters belong to the probability space and their standard errors were recovered using the Delta method (see Goldberger, 1991, p. 110 on this point).

[INSERT TABLES 4 AND 5 HERE]

The results confirm that central bank interventions are often associated with the jumps observed in the dynamics of exchange rates. For both currencies, almost all intervention variables are found to significantly influence the jump probability. For instance, the jump probability amounts to 0.76 when the Fed is intervening on the YEN/USD market (i.e. $Pr\left(\gamma_0+\gamma_1\right)$ in column "Intervention I" of Table 4), which is well above the constant probability observed when no intervention occurs, i.e. $Pr\left(\gamma_0\right)=0.10$. Since the occurrence of jumps is associated with an increase in exchange rate volatility, this implies that our results are fully consistent with the main conclusions reported by the core of the literature on the effects of central bank interventions. The distinction between individual interventions and coordinated ones also seems relevant, particularly for the DEM where the impact of coordinated interventions is less pronounced.

Interestingly, the introduction of a time-varying jump probability improves the fit of the model. Indeed, compared with estimation over the same period (see the first columns of Tables 4 and 5), it is found that the introduction of a time-varying jump probability lowers the observed excess kurtosis in the normalized residuals: for the DEM, the residuals do not exhibit excess kurtosis at the 1% nominal level while they do when a constant probability is assumed; for the YEN, the p-value lies around 5% with the time-varying probability while it was well below 1% before. Furthermore, the rejection level of the goodness-of-fit tests is much lower for the YEN with the time-varying probability.[19] Finally, a LR test clearly indicates that this model outperforms the constant jump probability framework. The improvement is less drastic for the DEM in the likelihood, information criteria and goodness-of-fit properties of the model; this is understandable given that the relevance of the distribution holds, even with a constant jump probability. For the DEM, since the jump only induces an increase in the variance, the interventions are found to exacerbate exchange rate volatility. For the YEN, the interventions tend to also influence the level of the exchange rate. These results confirm those of Beine, Bénassy and Lecourt (2002).

In the estimation results reported in Tables 4 and 5, the ζ parameter is not significant. This confirms that the presence of a long memory in the conditional mean is not a robust feature of

[19] Notice that the number of cells, g, has been adjusted according to the number of observations.

nominal exchange rates: it becomes insignificant once outliers are modelled appropriately (see also Granger and Hyung, 1999). By contrast, the long memory in the conditional variance is never questioned. However, while highly significant, the estimates of the d parameter are lower in the new model compared to the one with a constant jump probability. This supports the strong interaction between the jumps and the volatility persistence of exchange rates.

The column titled “Interventions II” reports a second specification, where we allow for impacts that depend on the direction of the interventions, i.e. whether the intervention is positive (the central bank is buying dollars) or negative (the central bank is selling dollars). Once again, there is no overlap in the values taken by the intervention variables: $x_{1,t}$ and $x_{2,t}$ refer respectively to individual purchases and individual sales by the FED; $x_{3,t}$ and $x_{4,t}$ refer respectively to individual purchases and individual sales by the Bundesbank or the BOJ; $x_{5,t}$ and $x_{6,t}$ refer respectively to simultaneous purchases and simultaneous sales by the two involved central banks.

Results suggest that buying or selling dollars does not equally influence the jump probability. For both the YEN and the DEM, the jump probability is higher when the central banks intervene in support of the dollar. Thus, there is an asymmetry which may be related to the special role of the dollar as an international currency. On the whole, this richer specification yields similar results with respect to the relevance of the distribution: compared to the constant probability model, one observes a decrease in the excess kurtosis and better goodness-of-fit properties.

5 Conclusion

In this paper, we estimate an ARFIMA-FIGARCH model for four major daily exchange rates against the US dollar over the 1980-1996 period. Special attention has been devoted to the choice of an appropriate distribution in the ML estimation procedure.

We find that an important number of outliers are responsible for the rejection of the normality assumption and that a significant part of these outliers are related to the direct central bank interventions in the foreign exchange markets. Consequently, we introduce a normal mixture distribution, the Bernoulli-normal, to account for these outliers.

Quite interestingly, mixtures of normal distributions are often considered as an alternative to long memory processes (see Diebold and Inoue, 1999 and Granger and Hyung, 1999 among others). The empirical results suggest that, in our case, the Bernoulli-normal distribution is not a substitute to the long memory in the conditional mean since, whatever the specification choice, we find no evidence of long memory in the conditional mean. By contrast, the long memory detected in the conditional variance is reduced when accounting for the jumps observed in the exchange rate dynamics.

Finally, and more importantly, the use of a time-varying jump probability explained by central bank interventions allows us to provide an economic interpretation of these jumps and improves the fit of the investigated series. Such a specification is obviously an alternative modelling strategy to long memory models with a normal distribution or to approaches excluding the outliers. Our results suggest that central bank interventions have increased exchange rate volatility, much in line with the findings in the empirical literature.

References

[1] Baillie, R.T., T. Bollerslev and H.O. Mikkelsen, 1996a, Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity, Journal of Econometrics 74, 3-30.

[2] Baillie, R.T., C.F. Chung and M.A. Tieslau, 1996b, Analyzing Inflation by the Fractionally Integrated ARFIMA-GARCH Model, Journal of Applied Econometrics 11, 23-40.

[3] Baillie, R.T. and W.P. Osterberg, 1997a, Why do Central Banks Intervene ?, Journal of International Money and Finance 16, 909-919.

[4] Baillie, R.T. and W.P Osterberg, 1997b, Central Bank Intervention and Risk in the Forward Market, Journal of International Economics 43, 483-497.

[5] Beine, M., A. Bénassy and C. Lecourt, 2002, Central Bank Intervention and Foreign Exchange Rates: New Evidence from FIGARCH Estimations, Journal of International Money and Finance 21, 115-144.

[6] Beine, M. and S. Laurent, 2001, Structural Change and Long Memory in Volatility: New Evidence from Daily Exchange Rates, in: Dunis, C., A. Timmerman and J. Moody (eds), Developments in Forecast Combination and Portfolio Choice, Wiley Series in Quantitative Analysis, Wiley, chap. 6, 145-157.

[7] Beine, M., S. Laurent and C. Lecourt, 2002, Accounting for Conditional Leptokurtosis and Closing Days Effects in FIGARCH Models of Daily Exchange Rates, Applied Financial Economics 12, 589-600.

[8] Bollerslev, T., 1986, Generalized Autoregressive Conditional Heteroskedasticity, Journal of Econometrics 31, 307-327.

[9] Bollerslev, T., 1987, A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return, Review of Economics and Statistics 69, 542-547.

[10] Bollerslev, T. and R.F. Engle, 1993, Common Persistence in Conditional Variances, Econometrica 61, 167-186.

[11] Bollerslev, T. and J.M. Wooldridge, 1992, Quasi-maximum Likelihood Estimation and Inference in Dynamic Models with Time-varying Covariances, Econometric Reviews 11, 143-172.

[12] Bonser-Neal, C. and G. Tanner, 1996, Central Bank Intervention and the Volatility of Foreign Exchange Rates: Evidence from the Options Market, Journal of International Money and Finance 15, 853-878.

[13] Boubel, A. and S. Laurent, 2001, Long-Run Volatility Dependencies in Intraday Data and Mixture of Normal Distributions, in: Dunis, C., A. Timmerman and J. Moody (eds), Developments in Forecast Combination and Portofolio Choice, Wiley Series in Quantitative Analysis, Wiley, chap. 7, 159-177.

[14] Brock, A., W.D. Dechert and J.A. Scheinkman, 1987, A Test for Independence Based on the Correlation Dimension, SSRJ Working Paper, 8762, Department of Economics, University of Wisconsin-Madison.

[15] Cheung, Y.W., 1993, Long Memory in Foreign Exchange Rates, Journal of Business and Economic Statistics 11, 93-101.

[16] Diebold, F.X. and A. Inoue, 1999, Long Memory and Structural Change, Manuscript, Department of Finance, Stern School, NYU.

[17] Dominguez, K.M., 1998, Central Bank Intervention and Exchange Rate Volatility, Journal of International Money and Finance 17, 161-190.

[18] Dominguez, K.M., 1999, The Market Microstructure of Central Bank Intervention. NBER Working paper 7337.

[19] Engle, R.F., 1982, Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation, Econometrica 50, 987-1008.

[20] Engle, R.F. and T. Bollerslev, 1986, Modeling the Persistence of Conditional Variances, Econometric Reviews 5, 1-50.

[21] Franses, H-P. and H. Ghijsels, 1999, Additive Outliers, GARCH and Forecasting Volatility, International Journal of Forecasting 15, 1-9.

[22] Granger, C.W.J. and R. Joyeux, 1980, An Introduction to Long-Memory Time Series Models and Fractional Differencing, Journal of Time Series Analysis 1, 15-29.

[23] Granger, C.W.J. and N. Hyung, 1999, Occasional Structural Breaks and Long Memory, UCSD Discussion Paper 99-14.

[24] Goldberger, A.S., 1991, A Course in Econometrics, Harvard University Press (Cambridge, Massachusetts).

[25] Hosking, J.R.M., 1981, Fractional Differencing, Biometrika 68, 165-176.

[26] Hotta, L.K and R.S. Tsay, 1998, Outliers in GARCH Process, Unpublished Manuscript.

[27] Kendall, M. and A. Stuart, 1967, The Advanced Theory of Statistics, Vol.2, Inference and Relationships, (Haffner, New York).

[28] Laurent, S. and J.-P. Peters, 2002, G@RCH 2.2: An Ox Package for Estimating and Forecasting Various ARCH Models, Journal of Economic Surveys 16, 445-485.

[29] Lecourt, C., 2000, Dépendance de Court et Long Terme des Rendements de Taux de Change, Economie et Prévision 5, 127-137.

[30] Neely, C.J., 1999, Target Zones and Conditional Volatility: the Role of Realignments, Journal of Empirical Finance 6, 177-192.

[31] Nelson, D.B. and C.Q. Cao, 1992, Inequality Constraints in the Univariate GARCH Model, Journal of Business and Economic Statistics 10, 229-235.

[32] Nieuwland, F.G.M, W.F.C. Verschoor and C.C.P. Wolff, 1994, Stochastic Trends and Jumps in EMS Exchange Rates, Journal of International Money and Finance 13, 699-727.

[33] Palm, F.C., 1996, GARCH Models of Volatility, in: Maddala, G.S. and C.R. Rao, Handbook of Statistics, 7, chap. 14, 209-240.

[34] Palm, F.C. and P.J.G. Vlaar, 1997, Simple Diagnostics Procedures for Modelling Financial Time Series, Allgemeines Statistisches Archiv 81, 85-101.

[35] Teyssière, G., 1997, Double Long-Memory Financial Time Series, Paper presented at the ESEM, Toulouse.

[36] Teyssière, G., 1998, Multivariate Long-Memory ARCH Modelling for High Frequency Foreign Exchange Rates, Proceedings of the High-Frequency Data in Finance, (Olsen & associates, Zurich).

[37] Tschernig, R., 1995, Long Memory in Foreign Exchange Rates Revisited, Journal of International Financial Markets, Institutions and Money 5, 53-78.

[38] Tse, Y.K., 1998, The Conditional Heteroskedasticity of the YEN-Dollar Exchange Rates, Journal of Applied Econometrics 13, 49-56.

[39] Vlaar, P.J.G., 1994, Exchange Rates and Risk Premia within the European Monetary System, Phd Thesis, Rijksuniversiteit Limburg (Maastricht, Netherlands).

[40] Vlaar, P.J.G. and F.C. Palm, 1993, The Message in Weekly Exchange Rates in the European Monetary System: Mean Reversion, Conditional Heteroskedasticity and Jumps, Journal of Business and Economic Statistics 11, 351-360.

[41] Weiss, A.A., 1986, Asymptotic Theory for ARCH Models: Estimation and Testing, Econometric Theory 2, 107-131.

[42] Wright, J.H., 1998, Rank Tests in GARCH Models, Mimeo, November.

Table 1: Order selection of the ARMA-FIGARCH model with Bernoulli-normal distribution using the Schwarz Bayesian Information Criterion

ARMA (n, s)-FIGARCH $(1, d, 1)$	DEM	FRF	GBP	YEN
$n = 0; s = 0$	2.1394	2.0564	**1.9598**	1.9700
$n = 0; s = 1$	2.1391	2.0563	1.9612	1.9705
$n = 0; s = 2$	2.1409	2.0581	1.9628	1.9724
$n = 1; s = 0$	**2.1388**	**2.0561**	1.9610	**1.9696**
$n = 1; s = 1$	2.1407	2.0582	1.9622	1.9712
$n = 1; s = 2$	2.1422	2.0588	1.9640	1.9731
$n = 2; s = 0$	2.1405	2.0580	1.9625	1.9699
$n = 2; s = 1$	2.1411	2.0583	1.9640	1.9706
$n = 2; s = 2$	2.1435	2.0607	1.9648	1.9724
ARMA (n^*, s^*)-FIGARCH (p, d, q)				
$p = 0; q = 0$	2.1485	2.0630	1.9630	1.9757
$p = 0; q = 1$	2.1504	2.0649	1.9650	1.9776
$p = 0; q = 2$	2.1524	2.0668	1.9669	1.9796
$p = 1; q = 0$	2.1405	2.0574	1.9615	1.9705
$p = 1; q = 1$	**2.1388**	2.0563	1.9598	1.9705
$p = 1; q = 2$	2.1408	2.0583	1.9617	1.9732
$p = 2; q = 0$	2.1532	**2.0546**	**1.9590**	**1.9704**
$p = 2; q = 1$	2.1395	2.0565	1.9609	1.9723
$p = 2; q = 2$	2.1414	2.0585	1.9628	1.9743

The upper panel gives the values of the Schwarz Bayesian Information Criterion (divided by T) across the various ARMA specifications using a FIGARCH $(1, d, 1)$ specification. The lower part reports the same statistics across various FIGARCH specifications using the AR and MA components selected in the first step.

Table 2: ARMA (n, s) - FIGARCH $(1, d, 1)$ models

	Bernoulli-normal distribution							
	DEM		FRF		GBP		YEN	
μ	0.0140		0.0102		-0.0123		0.0258	
	(1.381)		(1.114)		(-1.218)		(2.331)	
ψ_1	-0.0472		-0.0406		-		-0.0387	
	(-3.102)		(-2.647)				(-2.346)	
ω	-0.0246		-0.0134		-0.0285		-0.0415	
	(-0.806)		(-1.068)		(-5.585)		(-4.608)	
d	0.3206		0.2754		0.2608		0.2665	
	(5.652)		(7.322)		(7.769)		(7.033)	
β_1	0.5302		0.2028		0.1571		0.1912	
	(8.015)		(5.032)		(4.037)		(4.709)	
β_2	-		0.0895		0.0797		0.0608	
			(4.338)		(4.323)		(2.738)	
ϕ_1	0.2584		-		-		-	
	(5.533)							
λ	0.0651		0.0518		0.1481		0.1122	
	(1.312)		(1.981)		(4.085)		(3.521)	
τ	-0.1667		-0.0335		0.1018		-0.3068	
	(-0.966)		(-0.237)		(1.640)		(-3.692)	
δ^2	1.5762		2.2049		0.7466		1.2195	
	(1.709)		(2.122)		(4.571)		(3.893)	
b_3	-0.000		0.009		0.001		-0.049	
b_4	0.206	***	0.217	***	0.292	***	0.180	*
BDS(6)	-0.719		-0.825		-0.158		0.427	
S(5)	4.723		4.330		2.342		3.239	
P(70)	57.755		87.976	*	94.533		102.117	***
SC	2.139		2.054		1.959		1.970	
L_{Bern}	-4575.209		-4393.102		-4191.253		-4121.038	

Robust t-statistics are in parentheses. *,** and *** indicate rejection respectively at the 10%, 5% and 1% level (for the readability of the tables the *'s are not reported for the estimated parameters). *BDS(6)* corresponds to the t-stat of the BDS statistic with $m = 6$. *S(5)* is the nonparametric rank test for $l = 5$. The BDS and rank tests are computed on the normalized residuals. *P(70)* is the Pearson goodness-of-fit for 70 cells.

Table 3: Order selection of the ARMA-FIGARCH model with Bernoulli-normal distribution using the Schwarz Bayesian Information Criterion (period 1985-1995)

	ARMA (n, s)-FIGARCH $(1, d, 1)$			ARMA (n^*, s^*)-FIGARCH (p, d, q)	
Lags	YEN	DEM	Lags	YEN	DEM
$n = 0; s = 0$	**2.0405**	2.1394	$p = 0; q = 0$	2.0397	2.2278
$n = 0; s = 1$	2.0422	2.1391	$p = 0; q = 1$	2.0426	2.2306
$n = 0; s = 2$	2.0451	2.1409	$p = 0; q = 2$	-	-
$n = 1; s = 0$	2.0416	**2.1388**	$p = 1; q = 0$	**2.0384**	2.2249
$n = 1; s = 1$	2.0445	2.1407	$p = 1; q = 1$	2.0405	2.2242
$n = 1; s = 2$	2.0473	2.1422	$p = 1; q = 2$	2.0434	2.2270
$n = 2; s = 0$	2.0445	2.1405	$p = 2; q = 0$	2.0405	**2.2233**
$n = 2; s = 1$	-	2.1411	$p = 2; q = 1$	2.0433	2.2261
$n = 2; s = 2$	2.0473	2.1435	$p = 2; q = 2$	2.2290	2.2290

The left panel gives the values of the Schwarz Bayesian Information Criterion used for the selection of n^* and s^*; the right panel gives the values of the criterion with respect to the FIGARCH specification for n^* and s^*. - denotes that convergence was not achieved for this specific model.

Table 4: Time-varying jump probabilities for the YEN - ARMA $(0,0)$-FIGARCH $(1,d,0)$

	Constant Probability		Interventions I		Interventions II	
μ	-0.0008		-0.0020		0.0011	
	(-0.059)		(-0.147)		(0.070)	
ω	-0.0227		-0.0224		-0.0367	
	(-0.917)		(-1.321)		(-1.230)	
d	0.1879		0.1870		0.1872	
	(4.978)		(4.841)		(4.858)	
β_1	0.1263		0.1273		0.1249	
	(2.478)		(2.460)		(2.471)	
$Pr(\gamma_0)$	0.1214		0.1044		0.0997	
	(2.321)		(2.081)		(2.290)	
$Pr(\gamma_0+\gamma_1)$	-		0.7656		0.9987	
			(3.352)		(423.770)	
$Pr(\gamma_0+\gamma_2)$	-		0.3576		0.4705	
			(2.073)		(2.167)	
$Pr(\gamma_0+\gamma_3)$	-		0.4770		0.4717	
			(2.970)		(2.391)	
$Pr(\gamma_0+\gamma_4)$	-		-		0.1601	
					(0.781)	
$Pr(\gamma_0+\gamma_5)$	-		-		0.6671	
					(3.192)	
$Pr(\gamma_0+\gamma_6)$	-		-		0.2958	
					(1.963)	
τ	-0.2346		-0.2320		-0.2550	
	(-2.201)		(-2.332)		(-2.628)	
δ^2	1.3935		1.2870		1.3267	
	(2.837)		(2.989)		(3.353)	
b_3	-0.046		-0.071		-0.058	
b_4	0.229	***	0.182	**	0.187	**
BDS(6)	-1.077		-1.280		-1.232	
S(5)	7.172		5.080		11.182	**
P(50)	78.871	***	55.201	**	57.468	**
SC	2.038		2.028		2.035	
L_{Bern}	-2749.617		-2724.702		-2721.214	

$Pr(\gamma_0)$ stands for the constant jump probability while $Pr(\gamma_0+\gamma_i)$ stands for the jump probability associated to the case where $x_{i,t}=1$

Table 5: Time-varying jump probabilities for the DEM - ARMA $(1,0)$-FIGARCH $(2,d,0)$

	Constant Probability		Interventions I		Interventions II	
μ	-0.0267		-0.0324		-0.0293	
	(-1.890)		(-2.355)		(-1.970)	
ψ_1	-0.0373		-0.0405		-0.0409	
	(-1.973)		(-2.141)		(-2.151)	
ω	-0.0148		-0.0217		-0.0225	
	(-0.427)		(-0.431)		(-0.430)	
d	0.2302		0.2385		0.2352	
	(4.569)		(4.422)		(4.370)	
β_1	0.1983		0.2011		0.1997	
	(3.422)		(3.412)		(3.388)	
β_2	0.0786		0.0783		0.0784	
	(3.249)		(3.240)		(3.256)	
$Pr(\gamma_0)$	0.1086		0.0767		0.0797	
	(1.595)		(1.301)		(1.220)	
$Pr(\gamma_0+\gamma_1)$	-		0.3001		0.3778	
			(2.537)		(2.477)	
$Pr(\gamma_0+\gamma_2)$	-		0.2189		0.2164	
			(2.119)		(1.540)	
$Pr(\gamma_0+\gamma_3)$	-		0.3221		0.3459	
			(1.370)		(1.711)	
$Pr(\gamma_0+\gamma_4)$	-		-		0.1764	
					(1.933)	
$Pr(\gamma_0+\gamma_5)$	-		-		0.5911	
					(1.088)	
$Pr(\gamma_0+\gamma_6)$	-		-		0.2678	
					(1.237)	
τ	0.0078		0.0820		0.0407	
	(0.0712)		(0.733)		(0.299)	
δ^2	1.4686		1.5124		1.5121	
	(2.063)		(2.110)		(1.937)	
b_3	-0.039		-0.080	*	-0.064	
b_4	0.204	**	0.179	*	0.189	**
BDS(6)	-0.919		-1.077		-0.970	
S(5)	5.343		2.057		11.681	*
P(50)	36.559		36.310		44.912	
SC	2.223		2.224		2.231	
L_{Bern}	-3065.771		-3054.124		3052.615	

Note: see Table 4.