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Liquidity in the global currency market[☆]

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ABSTRACT

We study the liquidity of the global currency market by analyzing the price impact of trading volume. We analyze a decade of CLS intraday data representative of global foreign exchange (FX) trading by developing a refinement of the popular Amihud (2002) illiquidity measure that we call *realized Amihud*, which is the ratio between realized volatility and trading volume. Inversely related to market depth, price impact increases with transaction costs, money market stress, uncertainty, and risk aversion. Furthermore, we analyze whether and how liquidity begets price efficiency by looking at violations of the "triangular" no-arbitrage condition. We find that dollar-based currencies offer a lower trading impact supporting price efficiency.

1. Introduction

Since the demise of the post-war Bretton Woods system in the 1970s, the international financial system has witnessed growing capital mobility and wider movements of foreign exchange (FX) rates. In such a regime of floating FX rates and open economies, international investors pay close attention to the liquidity of currency markets. Thus, some natural questions arise: How should one measure FX market liquidity? What are its determinants? And, does liquidity support price efficiency?

We address these questions by analyzing high-frequency trading volume data from CLS Group, which are representative of the global currency market. To date, attempts to study global FX liquidity have been focused on one liquidity component: transaction cost. In addition to transaction cost, we conduct the first systematic study of another fundamental dimension of liquidity: the price impact of trading volume. To do this, we propose a

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refinement of the popular Amihud (2002) illiquidity measure.[1] First, we analyze the variation in trading impact over time, under market conditions, and across a relatively large set of FX rates representative of the global currency market. Second, we analyze the dynamic relation between market liquidity and price efficiency in terms of deviations from a no-arbitrage condition.

Analyzing FX liquidity is important for at least three reasons. First, the FX market is the world's largest financial market, with a daily traded volume of USD 6.6 trillion (Bank of International Settlements, 2019). It is often considered to be verging on market efficiency given its size and the prevalence of professional traders such as global dealers and sophisticated (financial) firms. However, an in-depth understanding of liquidity issues such as trading price impact and market depth is still missing. Second, FX rates are commonly traded in over-the-counter (OTC) markets, which are opaque and increasingly fragmented.[2] This OTC nature and the paucity of comprehensive trading volume data on a global scale has made it difficult to examine the relation between trading cost and price impact and between liquidity and price efficiency. Third, distressed markets, such as those during currency crises, are characterized by sudden drops in liquidity with adverse consequences for international investors and policy implementation, such as conducting (unconventional) monetary policy and FX interventions.

We proceed in two steps. First, we propose a new illiquidity measure called *realized Amihud* analogous to the Amihud (2002) illiquidity index, which is a common measure of trading price impact obtained as the ratio of absolute asset return to its dollar volume. The benefits brought by our measure are on two levels. On the one hand, it allows us to study how aggregate transaction volumes on a global scale impact currency prices. To do this, we access hourly data of FX trading volume from CLS Group (CLS) including 29 currency pairs (15 currencies) spanning November 2011 to September 2021.[3] On the other hand, we enhance the original Amihud measure by using high-frequency (intraday) return variations rather than the daily absolute return. In doing so, we gain a more accurate measurement of return volatility and a more precise estimate of price impact, which is observable and easy to calculate, confirming what was anticipated when the Amihud measure was proposed, namely that "there are finer and better measures of illiquidity ... that require a lot of microstructure data that are not available in many stock markets" (Amihud, 2002, p.32). Fortunately, the availability of granular, high-frequency data has increased significantly in recent decades.

Second, we analyze the relation between liquidity and price efficiency. Our premise is that a market is more efficient when an asset can be traded at a unique price.[4] On the contrary, violations of *the law of one price* indicate insufficient arbitrage activity and frictions such as illiquidity, undermining market efficiency. To do this, we analyze violations of the "triangular" no-arbitrage condition. That is, we compare the EURUSD rate with the rate derived by replicating the trade indirectly, for example by combining a EURGBP and a GBPUSD trade and six more "triplets."

Some new findings arise: the time-series analysis shows that the volume trading impact increases when the FX market is thinner. Despite the round-the-clock nature of FX trading, this occurs every day outside "London hours"[5] and when there are bank holidays in major financial centers such as the United States. The price impact on the global currency market is higher in distressed periods, such as the European sovereign debt crisis or during the COVID pandemic, and it systematically increases with transaction costs, money market strains, uncertainty, and risk aversion. For instance, a 1% increase in transaction costs is associated with a 0.20% increase in price impact. Furthermore, we utilize Electronic Broking Services (EBS) data on ultra-high-frequency orders and trades to compute benchmark measures of market illiquidity such as the effective spread and order flow price impact. We find that our illiquidity measure achieves very high correlations with the benchmark measures, suggesting that it is effective in sizing up the trading price impact in the global currency market.

Cross-sectionally, we quantify the price impact of trading in the most frequently traded pairs, such as EURUSD and USDJPY, and compare it with least traded (cross) rates, such as CADJPY and USDDKK, which are up to 200 times larger. On average, the price impact for an international investor is smallest if transactions are routed through U.S. dollars, whereas it increases with the euro, the yen, and the pound, sealing the dollar dominance and avoiding adverse price impacts (Somogyi, 2021). Economically, this translates into a larger dollar volume to induce a given FX rate movement. For example, to move one standard deviation of FX returns, it takes as much as $130 billion for EURUSD, while only less than half a billion for CADJPY. The overall picture that surfaces is that given the available liquidity, market participants deal with downward sloping demand curves for currencies (Hau et al., 2010), which are steeper outside the dollar.

One clear result that emerges in the second part of our paper is that price inefficiency systematically increases with illiquidity. Currencies benefiting from an average smaller *level* of trading impact are able to maintain higher price efficiency. For instance, the sensitivity to mispricing of an illiquid EURUSD triplet such as that based on the Norwegian krone is about four times higher than the liquid triplet bound to the yen. The comparison of the indirect currency pairs involving the euro and dollar to repli-

[1] By the end of April 2022, Amihud (2002) had been cited more than 10,800 times according to Google Scholar.

[2] See Chaboud et al. (2021) for a recent description of the FX market infrastructure.

[3] CLS operates the largest payment-versus-payment (PVP) settlement service in the world and CLS volume data covers around 50% of global FX turnover compared to Bank for International Settlements (BIS) triennial surveys (Cespa et al., 2022).

[4] Rather than informational efficiency, we refer to operational efficiency, whereby the market functioning allows traders to transact at a single price two or more positions referring to the same fundamental value and where price divergences are promptly restored without frictions.

[5] London hours are from 7 AM London open to 9 PM New York close, GMT.

cate the EURUSD rate highlights that the euro currencies, which are typically traded in thinner markets, concentrate the illiquidity frictions leading to arbitrage violations. In contrast, the dollar pairs serving as vehicle currencies are mostly disconnected from arbitrage deviations. The role played by the level of liquidity in establishing the strength of the link between mispricing and illiquidity becomes particularly evident when exploring this relation as it corresponds to events, such as bank holidays, that exogenously reduce the amount of trading activity.

Overall, the mispricing-illiquidity analysis suggests that dollar liquidity is crucial to preserving price efficiency as potential arbitrageurs outside the dollar deal with thinner markets and steeper demand curves. To gain more insight into the liquidity impact on price efficiency, we conduct three additional analyses: First, we examine the shift in the currency policy announced by the Swiss National Bank in 2015 that, unlike holidays, triggered unexpected and lasting effects. We clearly find that, in such an environment, both liquidity and price efficiency were persistently impaired, strengthening their interdependence. Second, we disentangle the liquidity-efficiency equilibrium relation on the EURUSD rate and highlight that the FX rates involving euros rather than dollars concentrate the illiquidity frictions linked to deviations from the triangular no-arbitrage parity, reiterating that the dollar liquidity most supports price efficiency. In this analysis, we take into account endogeneity issues stemming especially from the simultaneous dependence of the illiquidity of individual currencies on common global factors. To mitigate these issues, we apply a granular instrumental variable (GIV) approach in the spirit of Gabaix and Koijen (2020). Looking at the difference between the size- and equal-weighted average of daily liquidity measures, the GIV method benefits from the wide heterogeneity in the cross-section of currency pairs' liquidity and it captures idiosyncratic events of market illiquidity, thereby establishing the sensitivity of price efficiency to trading impact for a given currency. Third, we study the dynamic relation between mispricing and illiquidity in terms of their convergence to a long-term equilibrium. For this task, we adopt a vector error-correction model specification in the spirit of Granger (1986) and Johansen (2008). Our results indicate that illiquidity and mispricing are positively and strongly tied together in an equilibrium relation, and that it is, rather, a liquidity adjustment that restores the equilibrium after a shock hits the system.

We contribute to prior research on liquidity in financial markets. Most previous studies on the Amihud measure have mainly focused on stocks, proposing it as an illiquidity proxy capturing the price impact of trading[6] and analyzing pricing implications (see, e.g., Amihud and Noh, 2021). Although there is a growing literature on FX liquidity, the lack of transaction volume and flow data covering the entire currency market has led prior research to focus on trading volume and related liquidity issues solely for individual segments of the FX market. Specifically, the studies that have taken this direction have examined the FX interdealer segment for which volume and order flow data is available, since it mostly relies on electronic limit order book platforms such as EBS and Reuters/Refinitiv (see, e.g., Evans and Lyons, 2002; Bjønnes and Rime, 2005; Chaboud et al., 2007; Mancini et al., 2013).[7] Other papers analyze customers' flow for a specific bank (see, e.g., Evans and Lyons, 2006; Osler et al., 2011; Menkhoff et al., 2016). The analysis of market liquidity for the entire global FX market has been limited to estimates of transaction cost based on (indicative) bid and ask quotes (see, e.g., Huang and Masulis, 1999).[8]

Only with the recent availability of CLS data has research on global FX volume at relatively high frequencies (e.g., hourly or daily) become possible. Apart from CLS, the only source of global FX trading volume is the triennial survey of central banks conducted by the BIS providing a snapshot of FX market volume on a given day once every three years. Using CLS data, Fischer and Ranaldo (2011) look at global FX trading around central bank decisions, while Hasbrouck and Levich (2021) analyze the network structure of the FX spot market. Ranaldo and Somogyi (2021) study the information content of CLS flow data to highlight asymmetric information risk in FX markets. Cespa, Gargano, Riddiough and Sarno (2022) analyze the profitability of FX trading strategies exploiting the predictive ability of CLS volume. Somogyi (2021) proposes the strategic avoidance of price impact to explain the dollar dominance. Huang et al. (2022) analyze how dealers' financial constraints affect liquidity provision.

Hasbrouck and Levich (2019b) is the first study proposing an Amihud measure for FX trading. Specifically, the authors compute the classical Amihud proxy using all settlement submissions to CLS during April of 2010, 2013, and 2016. They show that the CLS-based measures are similar to their EBS-based counterparts, but not for those currencies that are not predominantly traded on EBS. We are the first to study global FX liquidity in relation to no-arbitrage conditions for a relatively large cross section of FX rates over a prolonged period. To do so, we propose a refinement of the Amihud (2002) measure that improves its accuracy and we study violations of the triangular no-arbitrage condition.[9]

This paper is organized as follows: Section 2 introduces the realized Amihud. Section 3 presents the data sets. The analysis of FX liquidity and liquidity in relation to price efficiency are provided in Section 4 and 5, respectively. Section 6 concludes the paper.

[6] See Foucault et al. (2013) (p 56–59) for an in-depth discussion of why the Amihud measure captures the price impact of trading.

[7] Among others, order flow is studied in Berger et al. (2008), Frömmel et al. (2008), Breedon and Ranaldo (2013), Evans (2002), Payne (2003), and Rime et al. (2010).

[8] Bid-ask spreads in FX spot markets are also studied in e.g. Bessembinder (1994), Bollerslev and Melvin (1994), Christiansen et al. (2011), Hartmann (1999), Hsieh and Kleidon (1996), and Karnaukh et al. (2015).

[9] Foucault et al. (2017) provide a microstructure analysis of the market maker's risk of trading at stale quotes tied to the triangular arbitrage in the interdealer segment while we analyze this arbitrage condition in connection with trading impact over a longer period and many currency pairs representative of the global FX market.

2. Liquidity measurement

As stressed in Kyle (1985), market liquidity is an elusive and multifaceted concept encompassing "tightness," "depth," and "resiliency." While tightness is measurable with proxies of transaction costs such as bid-ask spreads, it is more difficult to gauge market depth and resiliency, especially in decentralized and OTC frameworks such as the currency market. To do this, variables such as order flow and prices of financial securities representative of the entire market are needed. However, measuring order imbalances is difficult, as it requires intraday data on the signed flow of buy and sell market orders. The Amihud (2002) illiquidity measure circumvents this problem and it is widely used to approximate the price impact of transactions and Kyle's lambda (e.g., Chordia et al., 2009 and Foucault et al., 2013). Intuitively, the steeper the slope of the demand curve for a given supply of financial securities, the larger the price impact of a given volume size and the higher the related Amihud measure. Although trading volume and order imbalance are undoubtedly different concepts, they are correlated so that it is possible to relate the absolute value of price changes and the monetary value of the total amount traded over a given time interval, for example, daily or monthly (Hasbrouck, 2007).[10]

The original Amihud measure is computed as the ratio of the absolute asset return to its dollar volume on daily basis. This measure is very uncommon in currency markets, presumably due to the lack of data on transaction volume. Thus, our first innovation is to propose such an observable and easy-to-calculate measure using a decade of intraday data representative of the global currency market. The second innovation is to use a more accurate measure of price dispersion. To give it a conceptual framework, we develop a simple theory based on the marginal trader's reservation price deviating from the market price that generates trading volume. The theory is explained in more detail in Appendix A; here we summarize only the main features. First, the common latent factor creating volatility and volume is the heterogeneity in reservation prices originated from, for example, differences of opinion (Harris and Raviv, 1993), about liquidity needs or information sets as well as agents' discount factors (Glosten and Milgrom, 1985). Second, in each intradaily trading period, $i = 1, \ldots, I$, market depth plays a key role, as it represents the capacity of the market to allow large quantities to be exchanged at the intersection between demand and supply. Hence, a deep market leads to a small price impact. Third, daily FX volatility can be precisely estimated by exploiting the theory of realized variation (see Andersen et al., 2001, among others), which provides a remarkably precise non parametric measurement of the magnitude of reservation price variations aggregated across traders. In particular, following Barndorff-Nielsen and Shephard (2003), we define the *realized power variation* of order one (or realized absolute variation) as $RPV^{x|y} = \sum_{i=1}^{I} |r_i^{x|y}|$, where $r_i^{x|y}$ is the log-return on the FX rate $x|y$ in the ith sub-interval, with x as base currency and y as quote currency. Hence, we refine the original Amihud (2002) estimator as follows

$$A^{x|y} := \frac{RPV^{x|y}}{\nu^{x|y}}. \tag{1}$$

We call it *realized Amihud*. Intuitively, $A^{x|y}$ gauges the price impact of trading, that is the amount of volatility on a unit interval (as measured by $RPV^{x|y}$) associated with the traded "dollar" volume $\nu^{x|y} = \sum_{i=1}^{I} \nu_i^{x|y}$ in the same period. In other words, $A^{x|y}$ measures the amount of FX volatility associated with a unit of trading volume.[11] The theory developed in Appendix A shows that $A^{x|y}$ decreases with the market depth and the number of market participants. We validate our illiquidity measure by conducting a numerical analysis that shows (i) a consistent decrease of realized Amihud with market depth and traders' participation, (ii) its higher accuracy than the original Amihud measurement, and (iii) the negligible effect of bid-ask noise when the realized Amihud is calculated at frequencies not higher than a minute.[12]

Based on this intuitive framework, some empirical predictions can be laid down: (i) Volatility and volume comove as they are governed by a common latent factor. (ii) The trading impact is lower in periods of deep market and high participation. And (iii) currencies exchanged in deep markets systematically draw more trading volume and market participants featuring smaller trading impact.

3. Data and preliminary analysis

3.1. Data sets

Our empirical analysis relies on three data sets for FX spot transactions. First, trading volume data comes from CLS, which was launched in 2002 and is now the largest payment system for the settlement of foreign exchange transactions. By means of a payment-versus-payment mechanism, this infrastructure supports FX trading by reducing settlement risk and supporting market efficiency. For each hour of our sample period and each currency pair, we obtained directly from CLS the one-sided trading volume and number of trades.[13]

There are three main aspects of CLS data with related limitations: coverage, market participants, and settlement issues. First, CLS settles 18 currencies and 40 currency pairs, making it highly representative of the entire FX market. For instance, the currency pairs involving USD and EUR cover more than 85% and 94% of the total trading volume of the BIS triennial survey. However, some currencies, such

[10] Intuitively, the correlation between trading volume and order imbalance increases when orders take a well-defined direction, that is, they are predominantly buy or sell orders.

[11] Based on a different theoretical framework, Kyle and Obizhaeva (2016) propose a measure that relates dollar volume and volatility.

[12] The simulation analysis determines that the realized Amihud is about ten times more precise than the traditional Amihud and immune to bid-ask contamination unless volatility is computed at very high sampling frequencies shorter than 30 sec. These results are available in Internet Appendix.

[13] Although for a shorter sample period, the same data can also be obtained from Quandl.com, a financial and economic data provider.

as the renminbi and Russian ruble, are not included.[14] To maintain a balanced panel, we study 15 currencies (29 currency pairs) from the CLS original data over the period from November 1, 2011 to September 30, 2021.[15] Following the literature (e.g., Mancini et al., 2013), time is expressed in as GMT and we exclude observations between Friday 10PM and Sunday 10PM since only minimal trading activity is observed during these nonstandard hours.

Second, acting as the main settlement institution of the global FX trading, CLS gathers settlement instruction for transactions from a myriad of very different institutions encompassing major FX market-makers, international and locals, corporates and fund managers with different modus operandi. This diversity is emphasized by the OTC nature of the FX market and its two-tier and dealer-centric network, in which dealers manage a highly diverse client base. The result is a fragmented market in which transactions are executed using different electronic platforms, voice brokers, direct dealing, or other means. Combining all the settlement instructions, the CLS data loses the detailed information regarding various segments of the FX market and its participants. On the other hand, CLS offers an opportunity to analyze the FX market at an aggregate and global level, which is suitable for the purpose of this paper. But, to better understand the variety of participants, it is useful to briefly describe the structure of the CLS system.

In 2017, the core of CLS was composed of 60 settlement members, including the top 10 global FX dealers, and thousands of third parties (other banks, non-bank financial institutions, multinational corporations, and funds), that are customers of settlement members.[16] The total average daily traded volume submitted to CLS grew from more than USD 1.4 trillion in December 2014 to 1.83 trillion in September 2021 (the end of our sample period). The comparison between CLS and BIS figures indicates that the former was around 30% of the total daily volume recorded in the BIS triennial survey in both 2016 and 2019. However, after adjusting for the large fraction of BIS volume originated from interbank trading across desks and double-counted prime-brokered "give-up" trades, the CLS data should cover from 40% (Bech and Holden, 2019) to 50% (Cespa et al., 2022) of the global currency market.

Third, a settlement is not a transaction, and this has two important implications: On the one hand, CLS records the time of the transaction as if it had occurred when the first instruction was received. Therefore, the settlement time does not correspond exactly to trade time and can only occur afterwards. As discussed in Hasbrouck and Levich (2021), the settlement-trading correspondence is highly accurate for traded prices, quantities, and identities of trading parties, as settlement instructions result in irrevocable transfers of high value. What is more questionable is the discrepancy between settlement and trading *time*. Submission of settlement instructions lags the trade confirmation by a time period that can be very short or longer depending on the execution mechanism and types of market participants. By comparing the prevailing market price and settled price, Hasbrouck and Levich (2019b) estimate that 95.3% (81.3%) of the CLS trades can be matched within 30 sec, applying a price dispersion five (one) times the bid-ask spread. An internal study of CLS shows a fairly high proportion of individual settlement instructions included in the tick-by-tick prevailing bid-ask spread in the next few seconds or minutes.[17]

On the other hand, CLS receives electronic payment instructions for both sides of the trade, but it cannot identify which counterparty has actually initiated the transaction. In other terms, CLS volume is not signed in the sense that the active party (or initiator) and passive party of a transaction are not distinguishable. As explained in Ranaldo and Somogyi (2021), CLS determines the buy and sell side (supplier and demander of liquidity) by knowing the identity of market participants and inferring which banks play the role of liquidity providers. Since we utilize CLS volume and not CLS flows, this possible limitation of CLS data is irrelevant for this study.

Only a few papers have analyzed CLS volume data so far. First, Fischer and Ranaldo (2011) study the impact on global FX volume of central bank decisions using five aggregated currencies (e.g., all CLS-eligible currencies against the U.S. dollar, euro, yen, sterling, and Swiss franc), rather than currency pairs. Hasbrouck and Levich (2019b) analyze every CLS settlement instruction during April of 2010, 2013, and 2016. After calculating the Amihud index based on CLS data, they compare it to the corresponding estimates for the interdealer segment based on EBS and show high correlations for those currencies predominantly traded on EBS.[18] Using the same data, Hasbrouck and Levich (2021) analyze the network

[14] Starting in 2017, CLS has included the aggregation of matched FX trades for the offshore Chinese renminbi (CNH), Russian ruble (RUB), Turkish lira (TRY), and Polish zloty (PLN), each against the U.S. dollar (USD) and euro (EUR).

[15] The 29 currency pairs are: AUDJPY, AUDNZD, AUDUSD, CADJPY, EURAUD, EURCAD, EURCHF, EURDKK, EURGBP, EURJPY, EURNOK, EURSEK, EURUSD, GBPAUD, GBPCAD, GBPCHF, GBPJPY, GBPUSD, NZDUSD, USDCAD, USDCHF, USDDKK, USDHKD, USDJPY, USDMXN, USDNOK, USDSEK, USDSGD, and USDZAR. We also obtained CLS data for USDILS, USDKRW, EURHUF, and USDHUF. We discarded USDILS and USDKRW due to very infrequent trades. Since the Hungarian forint only joined CLS in 2015, HUF data are available only from November 7, 2015. Our raw CLS data with hourly trading volume contains a total of 1.821.000 observations.

[16] In 2020, there were 72 settlement members. Most of them are large multinational banks. Furthermore, there are over 25,000 "third party" clients of the settlement members, including other banks, funds, non bank financial institutions, and corporations.

[17] CLS randomly picked one day in March 2019 and seven currency pairs (AUDUSD, EURUSD, GBPUSD, NZDUSD, USDCAD, USDCHF, and USDJPY) for a total amount of 1,265,558 trades. The bid-ask spread was calculated by considering the highest bid and lowest ask quotes prevailing in the market using tick-by-tick data. The quoted spread has been widened by 20%. This exercise was replicated with two representative sources of FX spot trades. More than 87% (81%) of settlements fell within the spread within one (two) minute(s). The authors thank CLS for sharing this study and (anonymous) data.

[18] In addition to the traditional Amihud measure, Hasbrouck and Levich (2019b) also compute illiquidity ratios over fixed volume intervals and impact estimates based on bulk volume classification. In the online appendix (Hasbrouck and Levich, 2019a), they also offer an extensive analysis of CLS spot settlements per currency pairs, by year and size, and compare them with other data sources. Besides EBS, the other main interdealer platform is Thomson Reuters and some FX rates, for example, involving the British pound, are mainly traded on it.

structure of the FX spot market and provide evidence of a centrality premium, suggesting that dealers exercise bargaining power. Cespa et al. (2022) use FX volume data obtained from Quandl.com to perform an asset pricing analysis. Ranaldo and Somogyi (2021) analyze the heterogeneous price impact of CLS flows decomposed by market participants and FX asymmetric information risk premium. Somogyi (2021) shows that the dominance of the U.S. dollar in FX trading arises from the lower price impact (higher liquidity) offered by U.S. dollar currency pairs. Huang et al. (2022) analyze how market liquidity is affected by dealers' financial constraints.

The second data set is obtained from Olsen Financial Technologies, which is the standard source for academic research on intraday FX rates. Olsen collects streaming quotes from many dealing banks and multilateral platforms. By comprehensively compiling historical tick data, Olsen data are representative of the entire FX spot market, rather than specific segments such as the interdealer FX market, which is dominated by two electronic limit order markets: EBS and Reuters. For each minute of our sample period and each currency pair, we observe the following quotes: bid, ask, high, low, and mid-quotes. With this data, we can analyze at least four aspects of FX rates: (i) their movements at frequencies of one minute or lower; (ii) the realized volatility, realized power variation, or other measures of midquote return dispersion; (iii) the quoted bid-ask spread as a measure of transaction cost; and (iv) violations of triangular arbitrage conditions.

The third data set is obtained from EBS, which is the major interdealer trading platform for many currencies, including EURUSD, USDJPY, and EURCHF, which are studied more extensively in this paper. All our CLS FX rates are also traded in EBS, which operates an order-driven electronic trading system that unites buyers and sellers of spot FX around the globe on a pre-trade anonymous central limit order book. We access trade and order data for the entire year of 2016. The EBS data set is organized on a time slice basis, that is, at the end of each 100-millisecond we observe the total amount of trades, either buys or sells, during the time slice interval. Notice that EBS indicates whether a trade is buyer- or seller-initiated. In addition, we obtain trade price and volume (in millions of base currency). About order data, we observe the ten best "firm" bid and offer (or ask) quotes, capturing the depth of the book.

To visualize some emblematic behaviors, Panel a) of Fig. 1 reports the standardized hourly EBS and CLS volume on EURUSD on April 27, 2016. On that day, the Federal Reserve hosted the Federal Open Market Committee (FOMC) and the market was expecting an interest rate hike, which did not take place. Hence, the market reacted with a sharp and immediate depreciation of the dollar, which was subsequently reabsorbed. The main insight from Fig. 1 is that the trading volume in the interdealer segment (represented by EBS) followed a similar pattern to that of the global currency market (represented by CLS). Specifically, trading volume increased for both EBS and CLS during the announcement hour, with EBS volume reacting slightly more than CLS volume. In the subsequent hours, the volume of both EBS and CLS drastically reduced and it remained below the daily average in both cases. Panel b) of Fig. 1 shows the dynamics of FX rates and EBS volume around the announcement time at a 15-second frequency. Immediately after the announcement, the price dropped from 1.33 to 1.30 (−2.2% variation) and the volume did not react, thus indicating a sudden illiquidity episode. In the minutes after the announcement, trading volume and volatility increased enormously until 6:30PM, suggesting a significant disagreement in the reservation prices of each trader.

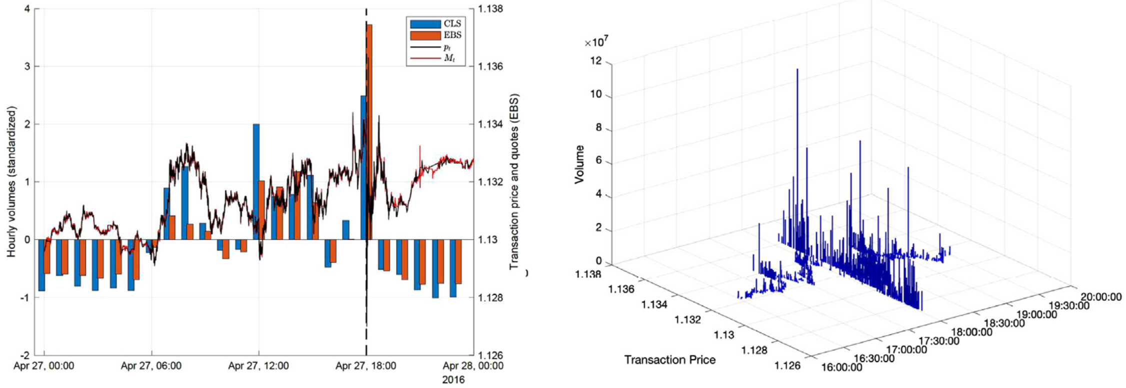
3.2. Descriptive analysis

We now highlight some abstract facts characterizing the times series of volume, volatility, and illiquidity measures. Some of these results have already been established for the interdealer segment, but others, especially those concerning global volumes, are new, although not all surprising. First, we look into intraday patterns and then we study the daily time series. Turning our attention to individual FX rates, Fig. 2 reports the hourly average share of total FX volume of the five most traded FX rates. Two considerations are worth noting. First, the five most liquid FX rates concentrate more than 70% of total global volume and, as expected, all of them involve the USD. However, their trading volume displays clearly different seasonal patterns suggestive of local effects in given geographical areas, consistent with the OTC segmented nature of FX markets. Between 12PM and 4PM, which are the hours during which Far Eastern markets are open, USDJPY covers around 30% of the total FX volume. AUDUSD contributes 15% of volume during these hours, while its market share strongly declines to 7% during the central hours of the day. EURUSD is by far the most traded FX rate during the "London hours", with a share above 30%.[19] A similar pattern also characterizes GBPUSD, with an average share ranging between 5% and 10%. Finally, USDCAD is mostly traded at the opening of business hours in North America, that is, between 12PM and 10PM, with an approximately 10% share of total volume.

Second, on an intraday scale trading volume follows the working time in each country or jurisdiction defining the currency pair. This means that, round-the-clock, the trading volume of the New Zealand dollar is the first to increase, followed by Asian, European, and American currencies. The natural consequence is that official bank holidays (studied in Section 5) significantly reduce the participation of local actors and therefore exchange volumes decline on those days.

Concerning the relation between volatility and volume, Fig. 3 shows that the averages of hourly volatility (RPV, in blue) and volume (in red) for EURUSD and USDJPY follow the same patterns. When FX volatility is high, so is volume, which points to the idea that the variation of traders' reservation prices is their common driver, as pre-

[19] The intense activity during London hours has already been discussed in the FX interdealer segment. For example, King et al. (2012) study six currency pairs and find that the average number of trades is higher during this time, while Ito and Hashimoto (2006) find that the bid-ask spread of EURUSD and USDJPY is smaller during the same time interval.



(a) EBS and CLS volume (b) Volume around the announcement

Fig. 1. Trading volume on April 27, 2016. Panel a) reports the hourly EBS volume (red bars) and CLS volume (blue bar). The volume series are those on the EURUSD rate; they are reported in deviation from the daily average. The dashed vertical line denotes the hour of the FOMC announcement. With a sampling at a 15-second frequency, Panel b) plots the EBS trading volume (z-axis) and the EURUSD rate (y-axis) centered around the FOMC announcement (18:00 GMT).

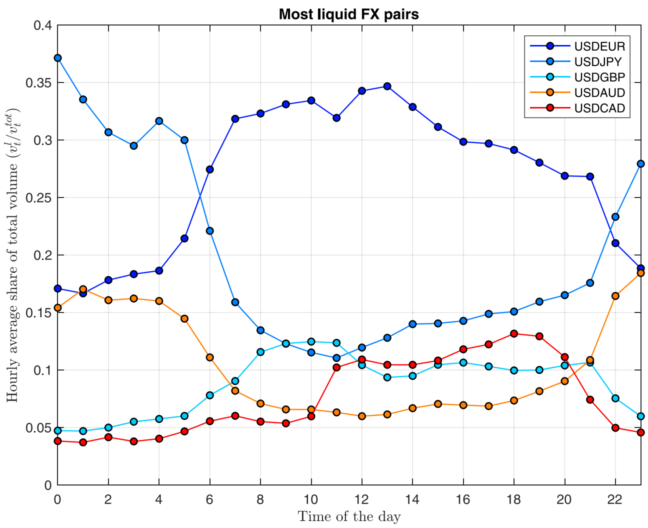


Fig. 2. Averages of hourly volume (relative to the total hourly volume) of the five most liquid FX rates involving currencies trading against USD, which are (in order) EUR, JPY, GBP, USD, and CAD.

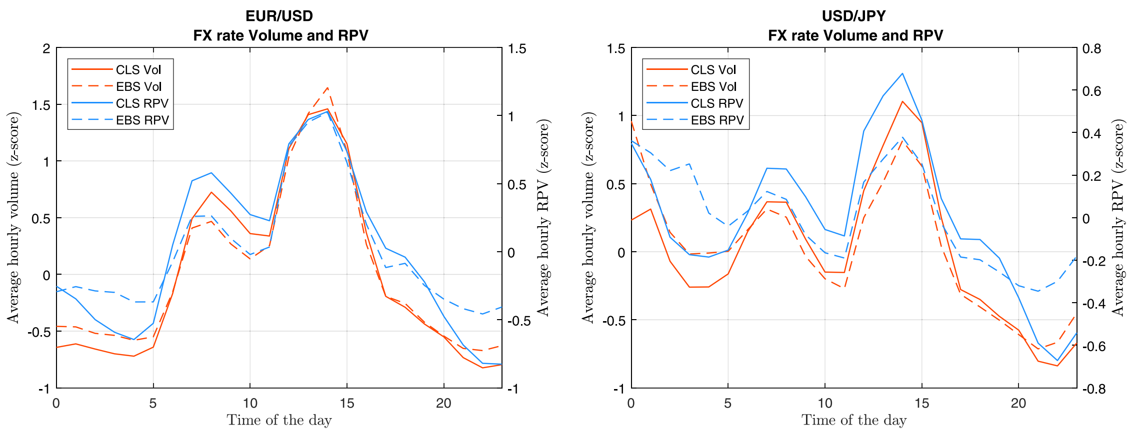


Fig. 3. Averages of hourly RPV and hourly trading volume. Hourly RPV and volume are based on the sum of absolute five-minute midquote returns and trading volume in each hour, respectively. In Panel a) EURUSD, in Panel b) USDJPY. Intraday patterns based on CLS and EBS data are depicted with continuous and dashed lines, respectively.

Table 1

Descriptive statistics of realized Amihud. Data are based on 2479 daily observations for each currency pair and the sample period spans from November 2011 to September 2021. Statistics for the volume-weighted averages of the largest currencies are reported at the bottom of the table. Realized Amihud values are rescaled by a factor of ten^{10}. Mean+ and Mean- indicate the sample average of realized Amihud with RPV computed with only positive (or negative) returns (i.e. RPV_+ and RPV_-). AMIVEST stands for inverse (realized) Amihud, indicating how many billions of USD are necessary on average to move the FX rate by one standard deviation.

	Mean	StDev	Skewness	Kurtosis	ACF(1)	ACF(10)	ACF(22)	Mean+	Mean-	AMIVEST
AUDJPY	0.2073	0.0691	1.4247	6.9613	0.6892	0.4937	0.3957	0.1038	0.1035	2.5031
AUDNZD	0.2770	0.1311	2.2755	12.5536	0.5453	0.2380	0.1138	0.1383	0.1387	2.0272
CADJPY	1.4200	0.8696	2.1989	10.8396	0.6367	0.4140	0.3620	0.7116	0.7084	0.4427
EURAUD	0.2352	0.0963	1.7676	8.8108	0.6125	0.4284	0.3339	0.1176	0.1177	2.2968
EURCAD	0.3556	0.1474	1.5827	8.3189	0.5474	0.3627	0.2953	0.1781	0.1775	1.5422
EURCHF	0.0482	0.0272	0.8687	4.7002	0.8051	0.6772	0.6017	0.0241	0.0241	16.994
EURDKK	0.0294	0.0245	6.0309	61.6440	0.4210	0.1077	0.0571	0.0147	0.0147	21.704
EURGBP	0.0418	0.0175	2.0041	10.8333	0.6949	0.5724	0.4395	0.0209	0.0209	12.999
EURJPY	0.0498	0.0214	2.1062	10.2412	0.7787	0.6728	0.5840	0.0249	0.0249	10.826
EURNOK	0.1095	0.0686	5.7235	56.2414	0.7762	0.4830	0.3261	0.0548	0.0547	5.1730
EURSEK	0.0809	0.0384	4.1333	32.5732	0.6463	0.3896	0.2186	0.0404	0.0404	6.6438
GBPAUD	0.7969	0.3995	2.0479	10.7954	0.6501	0.4070	0.2571	0.3984	0.3984	0.7279
GBPCAD	1.3354	0.8895	2.8134	17.5233	0.5145	0.3180	0.2185	0.6681	0.6673	0.5004
GBPCHF	1.2389	0.7212	1.9125	9.9916	0.4789	0.3612	0.2854	0.6192	0.6197	0.5131
GBPJPY	0.1760	0.0815	1.3436	5.3743	0.7657	0.6434	0.5602	0.0883	0.0878	3.3698
USDAUD	0.0212	0.0080	1.7553	10.8905	0.7796	0.6745	0.5709	0.0106	0.0106	25.103
USDCAD	0.0126	0.0051	3.2872	22.6649	0.3570	0.3062	0.3077	0.0063	0.0063	41.657
USDCHF	0.0411	0.0146	1.2832	6.5614	0.6196	0.4952	0.3868	0.0206	0.0205	12.889
USDDKK	1.1173	1.2703	11.1134	198.1386	0.1595	0.1441	0.1133	0.5602	0.5577	0.6839
USDEUR	0.0039	0.0012	1.2742	7.4840	0.6462	0.5623	0.4756	0.0020	0.0020	130.28
USDGBP	0.0129	0.0048	1.4955	8.7174	0.6869	0.5963	0.5160	0.0064	0.0064	41.764
USDHKD	0.0065	0.0049	4.8440	44.9048	0.5361	0.3034	0.2180	0.0033	0.0033	102.77
USDJPY	0.0071	0.0021	1.1535	5.5760	0.6861	0.5825	0.4684	0.0036	0.0035	71.568
USDMXP	0.0809	0.0424	2.7654	14.9388	0.6630	0.5840	0.4389	0.0405	0.0404	6.9998
USDNOK	0.3361	0.1741	2.7020	15.9239	0.5493	0.4244	0.2754	0.1683	0.1678	1.6976
USDNZD	0.0839	0.0333	2.4179	14.7224	0.6977	0.5403	0.3394	0.0420	0.0419	6.3545
USDSEK	0.2667	0.1306	2.0909	10.3985	0.5131	0.4185	0.3345	0.1335	0.1331	2.1251
USDSGD	0.0420	0.0161	2.1893	13.1635	0.6920	0.5442	0.4359	0.0210	0.0210	12.637
USDZAR	0.1767	0.0763	3.1696	25.8741	0.5553	0.4039	0.2875	0.0884	0.0884	3.0531
USD	0.0140	0.0041	1.8091	10.6712	0.6932	0.5926	0.4699	0.0070	0.0070	23.493
EUR	0.0238	0.0082	1.8908	10.2700	0.7091	0.5940	0.4607	0.0119	0.0119	21.757
JPY	0.0355	0.0117	1.5749	8.3640	0.7182	0.5677	0.4481	0.0178	0.0178	18.259
GBP	0.0587	0.0196	1.6630	9.3845	0.6521	0.5409	0.4183	0.0294	0.0294	8.8483

dicted by our theory. Furthermore, the comparison between CLS data (continuous lines) and EBS data (dashed lines) in Fig. 3 suggests that the global FX market and the FX interdealer segment follow the same systematic patterns, both in terms of trading volume and volatility. These results are in line with Huang and Masulis (1999), which shows that the bid-ask spread tends to be small when European and North American dealers are active in the market.[20]

The fact that the movement of volume is similar to that of volatility, as depicted in Fig. 3, implies that ex ante it is unclear how the depth of the global currency market and the related price impact evolve over time. All of this motivates an FX illiquidity measure capturing price impact in the spirit of Amihud (2002). To take the first step in this direction, Table 1 reports the descriptive statistics of the realized Amihud for the 29 FX pairs under investigation. Our attention is first caught by two statistics. First, the Amihud illiquidity measure is always positive and, in some cases, sizable, suggesting that traders in the currency market face downward-sloping demand curves (Hau, Massa and Peress, 2010); thus, trading higher volumes impacts FX rates more.

Second, these quantities reveal a large difference in price impact; for instance, the average realized Amihud of the three most illiquid currencies is more than 200 higher than the three most liquid. As expected, (cross) rates such as CADJPY and USDDKK that are considered illiquid feature the largest price impact, while the currency pairs such as EURUSD and USDJPY, which have the largest market share according to the BIS triennial survey, trigger the smallest impact. They also indicate the considerable trading impact that investors have to bear if they want to directly trade some currency pairs rather than indirectly exchanging them via “vehicle” currencies. From this point of view, it is quite surprising that, although relatively small, there is a part of the market that still trades those indirect currencies, as indicated by CLS and BIS data as well as in Hasbrouck and Levich (2019a).[21] Overall, the U.S. dollar appears to be the most attractive vehicle currency given that its exchange rates offer the lowest transaction impacts in terms of both average and volatility, even com-

[20] For a more detailed discussion, see Levich (2001, pp. 104–105).

[21] Future research should shed light on the characteristics of these market segments and whether their participants are retail clients with more rigid demand for such currency pairs.

pared to many of the exchange rates involving the euro. For example, the average (standard deviation) of the price impact when exchanging Canadian dollars for Japanese yens, rather than passing through USDCAD and USDJPY, is approximately 150 (300) times larger, suggesting that the dollar offers liquidity pooling with less (liquidity) risk. These findings are consistent with the idea that traders are better off using the dollar as the main vehicle currency to strategically avoid adverse price impacts (Somogyi, 2021).

An advantage offered by the use of high-frequency returns is the calculation of the realized volatility conditional on the positive or negative sign of the returns during the intraday time interval. In other words, the realized power variation can be broken down into two components: the sum of the absolute returns associated with the appreciation of the base currency with respect to the quote currency and that associated with the depreciation of the base currency, denoted respectively as $RPV_+ = \sum_{|r_i|} \mathcal{I}(r_i > 0)$ and $RPV_- = \sum_{|r_i|} \mathcal{I}(r_i < 0)$. The sample averages of the *signed* realized Amihud, denoted as $A+ = RPV_+/\nu$ and $A^- = RPV_-/\nu$, are shown in the third last and penultimate column of the Table 1, respectively. Intuitively, $A+$ and $A-$ measure the impact of trading associated with the appreciation or depreciation of the base currency against the quote currency. The general picture that emerges is that trading impact is substantially symmetrical, suggesting that the purchase of the U.S. dollar, which supports the demand for safe assets (Jiang et al., 2021), or the purchase of safe haven currencies such as the Swiss franc and Japanese yen (Ranaldo and Söderlind, 2010), which provide hedging benefits, does not imply a systematically higher trading impact and lower market depth. This seems to suggest that traders face an equilibrium curve that (in aggregate) is symmetric for positive and negative orders (see Eq. (A.1) in Appendix A).

Alternatively, one can measure market liquidity by looking at the trading volume associated with a standard deviation of FX returns. This is the inverse of the Amihud measure, also known as AMIVEST (e.g., Amihud, 2002). The rightmost column of Table 1 indicates that for highly liquid currencies like EURUSD or USDJPY it takes as much as \$130 or \$70 billion to move a standard deviation of FX returns. On the contrary, less than a billion dollars is sufficient for the same price impact for illiquid cross-currencies such as CADJPY, GBPAUD, GBPCAD, GBPCHF, or USDDKK.[22]

We also construct the aggregate realized Amihud measurements for the four main currencies (USD, EUR, GBP, and JPY). The indices are constructed as volume-weighted, cross-sectional averages of the individual realized Amihud measures with either USD, EUR, GBP, or JPY as a base (or quote) currency and are reported at the bottom of the Table 1.[23] On average, an international investor faces the lowest (largest) price impact (market depth) when trading with USD, followed by EUR, JPY, and GBP. This result is consistent with the dominant role of the dollar as the main reserve and vehicle currency. The cross-sectional aggregation also reduces the volatility and kurtosis of the series, making them more persistent than the realized Amihud for the individual rates. As for the individual FX rates, we note that the time series of realized Amihud are positively skewed, leptokurtic, and persistent.

To conclude the descriptive analysis, Fig. 4 reports the time series of the realized Amihud for USD and for the entire global FX market, which are obtained as a volume-weighted Paasche index, with November 1, 2011 as a base date. The evolution of the USD and global illiquidity indices displays similar patterns. It can be seen that liquidity deteriorates in times of stress such as the European sovereign debt crisis and the pandemic period. After the peak related to the European sovereign debt crisis, FX liquidity improved until the end of 2014, reaching approximately 50% of its original values. After January 2015, we notice a quick reversal to the same level of illiquidity values as in the beginning of the sample for both USD and global measures. Possible explanations could be (i) the shrinking interdealer market, coupled with the increasing application of internalization,[24] and (ii) regulatory pressure due, for example, to the Basel III and Dodd-Frank frameworks, which have since created shadow costs, thereby disincentivizing the dealer liquidity provision, as has been noted in other OTC and dealer markets (e.g., Adrian et al., 2017).

4. Liquidity analysis

4.1. Determinants of trading volume, volatility, and liquidity

Since volatility and trading volume are the key factors in the Amihud measure, we first examine how changes in daily trading volume correlate with changes in volatility (as measured by daily RPV) and other factors that have been shown to explain FX liquidity (measured as transaction costs; see, e.g., Karnaukh, Ranaldo and Söderlind, 2015) and trading activity in stock markets (e.g., Chordia et al., 2001). Although some of these variables are likely to be mutually endogenous, attention is directed towards finding novel correlation patterns pertaining to FX trading activity rather than causation. To this purpose, the 29 currency pairs are pooled together to examine whether daily FX volume is linked to changes in overall market conditions and its liquidity. More precisely, we consider the following linear regression model for daily trading volume

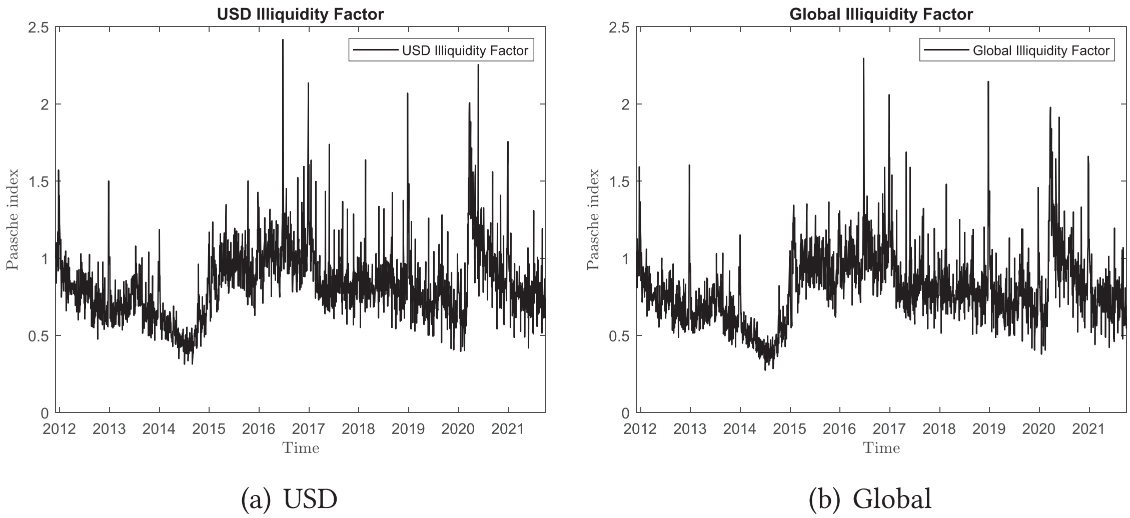
$$\nu_t = \beta_0 + \beta_1' x_t + \beta_2 \gamma_t + \beta_3' \delta_t + \beta_4 \nu_{t-1} + \varepsilon_t, \tag{2}$$

where x_t is a vector of regressors subsuming daily (realized) volatility, the relative bid-ask spread (BAS), the yield spread between the U.S. three-month Libor and T-bills (TED spread, a common proxy of funding strains), and the FX VIX (i.e., the JP Morgan Global FX implied volatility index, a proxy for uncertainty and global fear), which can be

[22] It is also interesting to note that, even for USDHKD, large volumes are needed for such a price movement. This is most likely due to its U.S. dollar peg rendering the movements of this FX rate more predictable and shielding it from volatility.

[23] In Section 5, we exploit volume-weighted, cross-sectional averages of the realized Amihud measure to construct instrumental variables.

[24] Internalization is the common and growing dealers' practice of temporarily warehousing the position originated from a client's transaction until it is offset against opposing client flow. See Chaboud et al. (2021) for a detailed discussion and evidence on this issue.



(a) USD (b) Global

Fig. 4. Time-series evolution of USD and Global FX illiquidity measures.

decomposed into time-varying uncertainty and risk aversion (Bekaert et al., 2013). Further, γ_t denotes a dollar appreciation dummy and δ_t includes day-of-the-week dummies. The same regression is then estimated for different dependent variables: volatility (RPV), the realized Amihud, and the relative bid-ask spread (BAS).[25]

The estimates of the regression coefficients are reported in Table 2. Some expected as well as novel patterns emerge from this regression analysis. First, the regression of volume on RPV (specification 1) confirms that FX trading volume increases with volatility. Furthermore, the same pattern arises when regressing RPV on volume (specification 2). This suggests that the common intraday volume-volatility relation shown in Fig. 3 also arises in their daily evolution. Second, transaction costs as measured by the bid-ask spread have an opposite effect on volume and volatility: they decrease with the former and increases with the latter, suggesting that larger transaction costs discourage trading activity and increase price dispersion. Third, uncertainty and global fear (gauged by FX VIX) also disincentivize trading activity and increase volatility. Fourth, when decoupling risk aversion from uncertainty, we observe that uncertainty has a negative impact on trading volume, while risk aversion increases with volume, perhaps due to investors' propensity to adjust their portfolio positions when becoming more risk-averse. As expected, volatility increases with uncertainty and risk aversion. Finally, trading volume and volatility follow an inverted U-shape relation across weekdays, that is, they tend to be larger in the middle of the week.

We now turn our attention to the regression results of the daily realized Amihud, which are exhibited in column (3) of Table 2. Overall, we find that the realized Amihud illiquidity measure increases with transaction costs, the TED spread, and implied volatility as well as indicators of risk aversion and uncertainty. In addition to being statistically significant, these results are economically meaningful. For instance, an increase in transaction costs of 1% is associated (ceteris paribus) with an increase in illiquidity of 0.20%. Overall, our results appear consistent with standard empirical predictions of microstructure theories, as they suggest that (i) the two main dimensions of market illiquidity, that is, transaction cost and price impact, are positively related and (ii) the price impact of trading gets larger with money market strains, uncertainty, and risk aversion.

To shed light on the interlinkage between two main dimensions of liquidity, that is, transaction costs and trading price impact, it is interesting to compare the results exhibited in columns (3) and (4) of Table 2. Compared to transaction costs, the trading impact reacts more to funding strains (TED) and uncertainty and is especially large at the beginning of the week. Also, the positive autocorrelation of trading price impact is less strong than that of transaction costs. The trading impact does not appear to be larger when the U.S. dollar appreciates, and this result is consistent with the overall symmetrical price impact shown in Table 1.

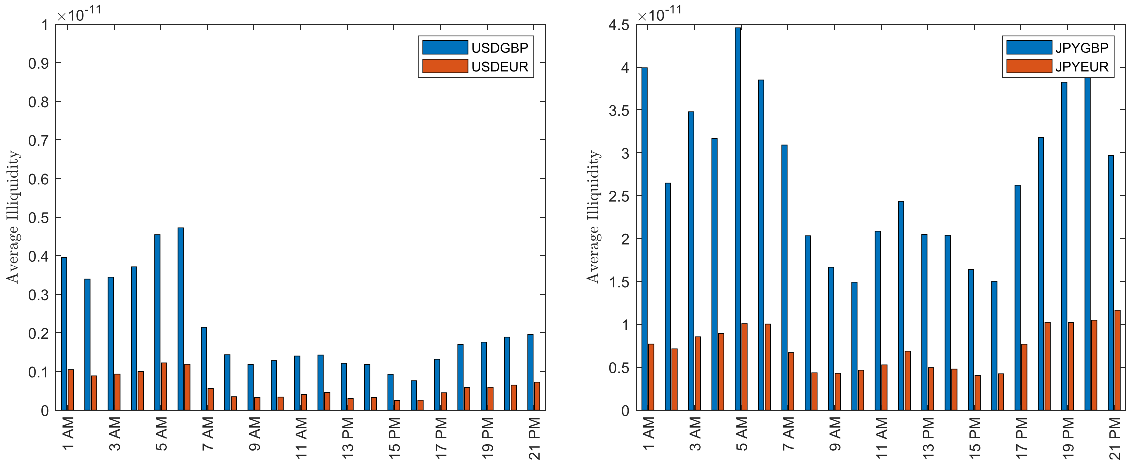
4.2. How well does the realized Amihud measure illiquidity?

As discussed above, our empirical prediction is that the price impact of trading decreases with market depth and the number of active traders. The visual inspection of Fig. 5 representing the intraday patterns of the realized Amihud, shows that the trading impact is smaller during London hours, that is, when the FX market is deep and populated by active traders. Regardless of which currency is considered, the price impact abruptly decreases at the opening of the European markets and is minimal when both European and American markets are jointly open. After 8PM the illiquidity grows again and is maximal during the night hours. The volume impact in the most and least liquid mo-

[25] After ensuring that the time series were stationary, this analysis was carried out in both levels and changes. We show the results of the former here, while the latter provides fully consistent results and is available on request.

Table 2
Regressions of volume, volatility (RPV), realized Amihud, and bid-ask spread. Volume and RPV are the daily trading volume and daily RPV, respectively; the realized Amihud is the ratio between daily RPV and daily volume; and the bid-ask spread is the daily average of one-minute spreads. Data are based on 2479 daily observations for each currency pair and the sample period spans November 2011 to September 2021. The *t*-statistics are in parentheses and the standard errors are robust to heteroskedasticity and autocorrelation in the residuals. The superscripts *a*, *b*, and *c* indicate significance at 1%, 5%, and 10% significance level, respectively.

	Volume		RPV		Realized Amihud		Relative BAS	
	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)	(4a)	(4b)
Volume			0.0773[a]	0.0763[a]			−0.0181[a]	−0.0182[a]
			(12.23)	(12.50)			(−10.47)	(−10.46)
RPV	0.0491[a]	0.0496[a]					0.1032[a]	0.1033[a]
	(17.60)	(17.91)					(10.70)	(11.07)
Relative BAS	−0.0585[a]	−0.0587[a]	0.3457[a]	0.3462[a]	0.0020[a]	0.0020[a]		
	(−22.89)	(−22.90)	(11.10)	(11.22)	(10.91)	(11.31)		
TED	−0.0108[b]	−0.0111[b]	0.0048	0.0072	0.0025[a]	0.0024[a]	−0.0190[a]	−0.0201[a]
	(−2.45)	(−2.47)	(0.30)	(0.43)	(8.30)	(7.91)	(−3.10)	(−3.21)
VXY	−0.0722[c]		0.9386[a]		0.0418[a]		−0.1152[a]	
	(−1.66)		(7.88)		(14.41)		(−2.62)	
UNC		−0.0015		0.0135		0.0023[a]		−0.0035
		(−0.48)		(1.29)		(11.19)		(−0.82)
RA		−0.0071[b]		0.1058[a]		0.0024[a]		−0.0068
		(−2.11)		(5.93)		(11.74)		(−1.23)
USD	0.1266	0.1354	0.6144[b]	0.4635[c]	−0.0055	−0.0063	0.1156	0.1201
	(0.99)	(1.06)	(2.12)	(1.67)	(−0.95)	(−1.08)	(1.20)	(1.28)
Monday	−3.6037[a]	−3.5993[a]	−4.1860[a]	−4.2187[a]	0.1682[a]	0.1680[a]	−3.3579[a]	−3.3548[a]
	(−17.38)	(−17.36)	(−8.51)	(−8.70)	(16.78)	(16.7408)	(−19.47)	(−19.72)
Tuesday	1.7320[a]	1.7336[a]	−0.8763[b]	−0.8950[b]	−0.0758[a]	−0.0757[a]	−0.5753[a]	−0.5747[a]
	(9.24)	(9.26)	(−2.23)	(−2.27)	(−8.54)	(−8.53)	(−4.54)	(−4.55)
Thursday	−0.2992	−0.2927	−0.1841	−0.2835	0.0221[a]	0.0218[a]	−0.1351	−0.1303
	(−1.61)	(−1.58)	(−0.38)	(−0.60)	(2.66)	(2.63)	(−0.91)	(−0.88)
Friday	−1.1475[a]	−1.1420[a]	−6.1259[a]	−6.1535[a]	0.0068	0.0068	0.9458[a]	0.9501[a]
	(−6.02)	(−5.98)	(−13.01)	(−13.10)	(0.88)	(0.88)	(5.83)	(5.88)
Lagged Dep.	0.9309[a]	0.9310[a]	0.6985[a]	0.6912[a]	0.4621[a]	0.4614[a]	0.8636[a]	0.8634[a]
	(151.73)	(151.74)	(27.81)	(27.29)	(16.86)	(16.85)	(88.28)	(87.92)
Constant	2.0836[a]	1.6043[a]	−3.6094[b]	3.0184[a]	−0.0030	0.1828[a]	1.7015[a]	1.0436[a]
	(5.58)	(6.18)	(−2.41)	(3.50)	(−0.16)	(11.04)	(3.63)	(3.32)
R^2	0.89	0.89	0.79	0.79	0.35	0.35	0.93	0.93



(a) USD (b) JPY

Fig. 5. Hourly averages of realized Amihud from 1AM until 9PM for GBP (in blue) and EUR (in red) trading against USD (lhs) and JPY (rhs). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 3
Correlation matrix for illiquidity measures for the EURUSD rate. Pearson (Spearman) correlations are reported in the lower (upper) triangular portion of the table. The data set comes from EBS and ranges from January 1, 2016 to December 31, 2016. *A*, denotes the realized Amihud, *BAS* the bid-ask spread, *EC* the effective cost, *CS* the Corwin-Schultz spread estimator, *R* the Roll measure, λ the trade-by-trade price impact coefficient, A^* the classic low-frequency Amihud measure computed with the absolute value of daily log-return, and A^{**} an alternative low-frequency Amihud measure computed with daily range at the numerator.

	A	BAS	EC	CS	R	λ	A^*	A^{**}
A	1.0000	0.7885	0.6680	0.3877	0.7068	0.9115	−0.0671	0.2622
BAS	0.9080	1.0000	0.7944	0.5247	0.6594	0.7923	−0.0115	0.1682
EC	0.8712	0.9128	1.0000	0.5701	0.8901	0.7455	0.0687	0.1843
CS	0.5460	0.6335	0.6570	1.0000	0.4730	0.4285	−0.0784	−0.0994
R	0.6791	0.5696	0.7950	0.4361	1.0000	0.7690	0.0289	0.1931
λ	0.9041	0.7759	0.8332	0.5583	0.7523	1.0000	−0.0304	0.2510
A^*	0.4005	0.4326	0.3683	−0.0873	0.1933	0.2138	1.0000	0.5881
A^{**}	0.5390	0.5204	0.4567	-0.0679	0.2928	0.3284	0.9248	1.0000

ments of the day can be more than five times stronger, and this also applies to the most liquid currency (EURUSD; on the left-hand side of Fig. 5). However, this difference is less pronounced for currencies involving the yen (EURJPY and GBPJPY; on the right-hand side of Fig. 5) given that trading activity in the Asian currencies is supported more by local market participants during the early morning hours than during the rest of the day.

So far, we have analyzed FX illiquidity on a global scale. Now, we ask whether our FX illiquidity measure is positively correlated with other illiquidity proxies in the FX *interdealer* segment. We focus on EURUSD since it is primarily traded on the EBS interdealer trading platform.[26] In the spirit of Hasbrouck (2009), we analyze correlations between daily illiquidity measures.[27] More specifically, we compute correlations of the realized Amihud (denoted as *A*) with the following proxies: relative quoted spread (i.e., quoted spread, or ask minus bid quotes, divided by midquote; denoted as *BAS*); effective cost, denoted as *EC* (i.e., the absolute value of the difference between transaction price and midquote); high-low estimate of the effective spread (denoted as *CS*) (Corwin and Schultz, 2012); cost estimates implied by the Roll model (Roll, 1984), which are computed as the autocovariance between consecutive price changes with negative estimates set to zero (denoted as *R*); trade-by-trade order flow price impact (denoted as λ); the traditional Amihud measure (i.e., daily return in absolute value over trading volume; denoted as A^*); and a new version of the traditional Amihud measure that replaces the daily absolute return with the daily high-low range as the numerator (denoted as A^{**}).[28]

Table 3 shows the correlation matrix for illiquidity measures and delivers three main messages: First, realized Amihud is positively correlated (Pearson) with intraday illiquidity proxies based on EBS data. In particular, it is highly correlated with the effective cost and order flow price impact, which are generally used as the benchmark measures of transaction cost and trading price impact, respectively (e.g., Hasbrouck, 2009). Overall, this finding corroborates the idea that the realized Amihud effectively captures the price impact of trading volume. Second, the realized Amihud index is positively correlated with the traditional low-frequency Amihud indicator. On the one hand, the volatility signal is less accurate when using low frequency (daily) data, thus resulting in smaller correlation with the other proxies of FX illiquidity compared with the realized Amihud. On the other hand, one could obtain estimates of FX trading price impact even approximating volatility with daily absolute returns (as in the traditional Amihud indicator), rather than gauging it with more accurate high-frequency measures (RPV). This also applies to the other new version of the low-frequency Amihud index we have proposed, whose numerator is the daily range. Actually, the range-based version seems to offer higher correlations with the intraday benchmarks than the traditional Amihud index. Third, the realized Amihud is correlated (with the expected sign) with other low-frequency estimates of effective spread, in particular the Roll measure.[29] Overall, we find that realized Amihud is highly and positively correlated with other illiquidity proxies, suggesting that it is effective in measuring FX illiquidity and, in particular, the price impact of trading. The Spearman rank correlations mostly confirm these results, although the magnitudes are somewhat smaller.

To conclude the liquidity analysis, we explore the relation between the two main measures of illiquidity, namely the realized Amihud and the relative bid-ask spread.

Unlike the previous analysis in Table 2 on the contemporaneous relation between them, here we take the two liquidity measures as dependent variables in a vector au-

[26] The analysis of USDJPY, which is also predominantly traded on EBS, provides similar results.

[27] Further details on the construction of the FX illiquidity measure are presented in the Internet Appendix.

[28] The daily relative quoted and effective spreads are average values of intraday data snapped at 100-millisecond intervals. Using the quoted spread gives very similar results. The Corwin-Schultz spread estimator is computed for each day using hourly maximum and minimum prices. A daily measure is then extracted by averaging out the 23 hourly spread estimates. Negative estimates are set to zero. Furthermore, the correlation coefficients of the realized Amihud with the order flow price impact based on five- or one-minute intervals are similar to the trade-by-trade order flow, although slightly smaller.

[29] The same picture holds when the Roll estimator is augmented with the Gibbs sampling method proposed by Hasbrouck (2009).

Table 4
VAR-X estimates. The dependent variables are realized Amihud (A_t) and relative bid-ask spread (BAS_t). Data include 2479 daily observations for all currency pairs and are measured in logs. The superscripts *a*, *b*, and *c* indicate significance at 1%, 5%, and 10% significance level respectively. The standard errors are robust to heteroskedasticity and autocorrelation in the residuals. The R^2 are 46% for the realized Amihud equation, and 94% for the relative BAS equation.

	BAS_{t-1}	A_{t-1}	USD$_t$	TED$_t$	VXY$_t$	Monday	Tuesday	Thursday	Friday	Const.
A_t	0.044^a	0.589^a	−0.009^a	0.025^a	0.314^a	0.149^a	−0.084^a	0.0226^a	0.002	−0.971^a
BAS_t	0.965^a	−0.007	0.002	−0.004	0.044^a	−0.109^a	−0.011^a	−0.005	0.012^a	0.064^a

toregressive (VAR) model, with the same exogenous variables used in regression (2), that is, USD, VIX, TED, and day-of-week dummy variables. The estimation results are presented in Table 4. The new interesting finding is that the yesterday's spread significantly and positively impacts today's realized Ahimud, but not vice versa. This suggests that an increase in the bid-ask spread leads to a subsequent increase in trading impact. One possible explanation for this is that the way dealers set their quotes (and bid-ask spread) subsequently determines the market depth and trading impact.

5. Liquidity and price efficiency

We now highlight whether and how liquidity relates to price efficiency assuming that FX rates are more efficient if they do not violate the triangular arbitrage condition. By triangular no-arbitrage parity, it must hold that $r^{x|y} = \tilde{r}^{x|y;z}$, where $\tilde{r}^{x|y;z} = r^{x|z} + r^{z|y}$ is the *synthetic* return on the $x|y$, passing through trading on a third currency, z, with FX returns $r^{x|z}$ and $r^{z|y}$. The rationale is that an illiquid currency enduring a larger price impact prevents traders from acting promptly and efficiently so as to restore the no-arbitrage parity. Using mid-quote returns based on Olsen data snapped every minute, we consider the absolute no-arbitrage violations (*pricing errors*) on the *i*th intradaily subinterval, defined as $pe_i^{x|y;z} = \tilde{r}_i^{x|y;z} - r_i^{x|y}$. Then, we obtain a simple measure of mispricing called RPVE$^{x|y;z}$ by summing these pricing errors in absolute value over hourly or longer periods

$$RPVE^{x|y;z} = \sum_{i=1}^{I} |pe_i^{x|y;z}|. \tag{3}$$

It should be stressed that the $RPVE^{x|y;z}$ indicator is not meant to precisely measure the actual arbitrageurs' profits, but to broadly capture the tendency to deviate from the law of one price. Setting EURUSD as the direct currency pair $x|y$, we examine the dependence between the variability of pricing errors ($RPVE^{x|y;z}$) and the average illiquidity of the two indirect FX rates forming the triangular arbitrage, computed as $\overline{A}^{x|y;z} = (A^{x|z} + A^{z|y})/2$, where $A^{x|z} = \frac{RPV^{x|z}}{\nu^{x|z}}$ and $A^{z|y} = \frac{RPV^{z|y}}{\nu^{z|y}}$ represent the realized Amihud of $x|z$ and $z|y$. For instance, for the triplet EURUSD, EURGBP, and GBPUSD, we consider the cumulative absolute pricing errors between the direct (EURUSD) and synthetic rates (via EURGBP and GBPUSD), and the average realized Amihud measures of the two indirect FX rates (EURGBP and GBPUSD).

Figure 6 plots the monthly cumulative pricing error variation ($RPVE^{x|y;z}$) against average trading impact ($\overline{A}_t^{x|y;z}$) for the EURUSD tied to USDJPY-EURJPY rates (red crosses) and to USDNOK-EURNOK rates (blue circles). The figure provides two insights: First, it clearly displays a positive relation between mispricing and illiquidity, suggesting that as the trading volume impact increases, deviations from a no-arbitrage condition also increase. In contrast, when traders including potential arbitrageurs face deeper markets, price efficiency increases. Second, Fig. 6 compares the most liquid triplet (based on JPY) to the least liquid one (NOK).[30] The steeper (flatter) slope related to the NOK (JPY) triplet suggests that less liquid currencies feature a stronger dependence between mispricing and illiquidity compared with liquid ones. Overall, Fig. 6 provides visual evidence that the *level* of liquidity is important in establishing the strength of the link between illiquidity and mispricing. Apparently, potential arbitrageurs trading less liquid currencies face larger price impacts, suggestive of steeper demand curves.

5.1. Holidays

A simple way to validate the idea that the liquidity level is important in determining the mispricing-illiquidity relation is to observe how it changes when the liquidity level is exogenously reduced by major holidays, such as official U.S. bank holidays. The illustration of this effect can be seen in Fig. 7, which shows the relation between illiquidity and mispricing on U.S. bank holidays. The visible shift of the relation to the right indicates a decrease in liquidity and a thinner market.

To understand whether liquidity and price efficiency are systematically related, we regress our mispricing measure ($RPVE^{x|y;z}$) on the average realized Amihud ($\overline{A}^{x|y;z}$) and then we extend this simple OLS regression as follows

$$RPVE_t^{x|y;z} = \alpha + \beta \bar{A}_t^{x|y;z} + \delta_\alpha H_t + \delta_\beta \bar{A}_t^{x|y;z} H_t + \varepsilon_t, \tag{4}$$

where H_t is the dummy variable equal to 1 if there is a bank holiday in the United States and we interact this dummy variable with the illiquidity measure. This simple analysis could indicate whether (i) a deterioration in liquidity is systematically associated with more inefficient pricing, and this would be captured by a significantly positive parameter β; (ii) arbitrage deviations are less frequent when reduced trading activity is expected (due to holidays), and this would be represented by a significantly negative parameter δ_α; and (iii) whether the connection between liquidity and price efficiency in an ex-ante less

[30] Economically, the realized Amihud in Table 1 averaged across the three currency pairs forming the JPY and NOK triplets is 0.02 and 0.15, respectively.

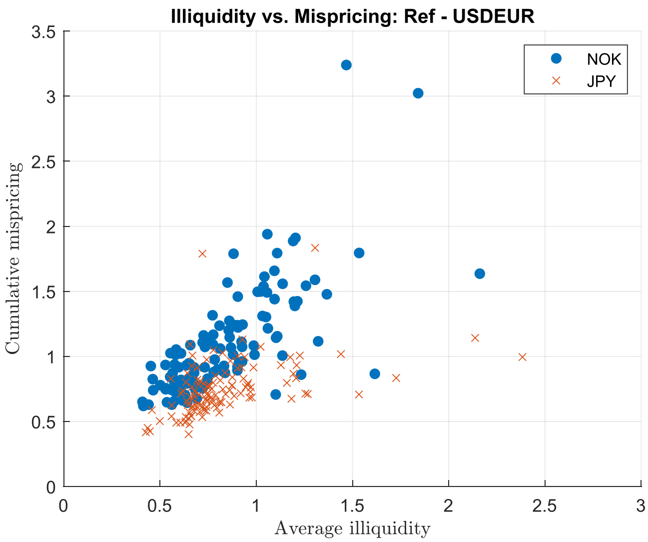
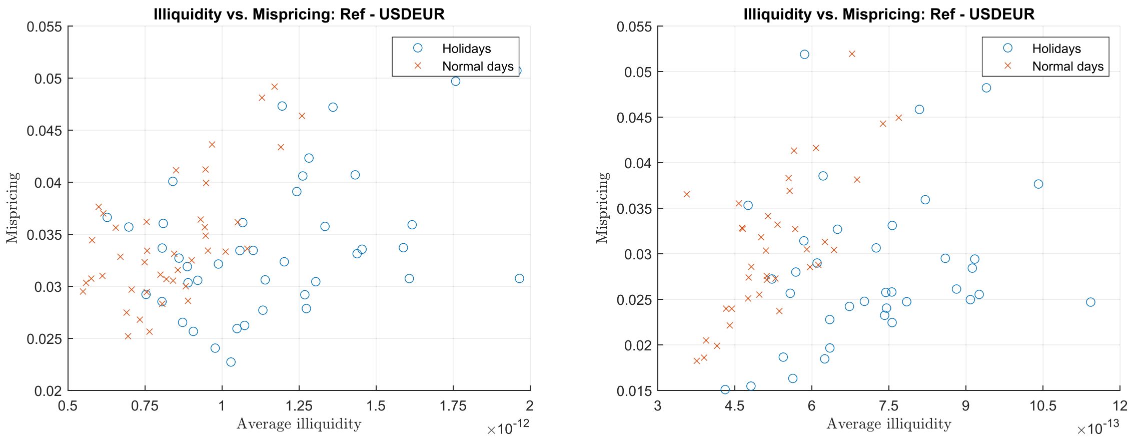


Fig. 6. Monthly cumulative pricing error variation ($\mathrm{RPVE}^{x|y;z}$) against monthly average illiquidity, $\overline{A}_t^{x|y;z}$. The liquidity time series for each currency pair are indexed to the first observation for better visualization.



(a) GBP   (b) JPY

Fig. 7. The figure shows the relation between illiquidity and mispricing on days when there are bank holidays in the United States (blue circles) vis-a-vis normal days (red crosses). Normal days are computed as the average values of the five days before a given holiday in the United States.

liquid environment (due to holidays) changes, and this would show up a parameter δ_β significantly different from zero.

The results presented in Table 5 support these predictions. Looking at the baseline specification, the parameter β is found to be positive in all specifications and for all currencies, thus confirming the graphical illustration in Fig. 6. The largest values of β are found for SEK and NOK, for which an increase in illiquidity is associated with a larger impact on mispricing compared to other currencies. Given that these two currencies are among the most illiquid in our sample, this result reaffirms what had emerged earlier, that is, the liquidity level appears important in the way liquidity begets price efficiency. Ceteris paribus, arbitrage violations appear to be more strongly linked to volume price impacts in illiquid currencies, consistent with the idea of steeper downward sloping demand curves faced by potential arbitrageurs dealing with illiquid currencies.

To conclude this regression analysis, we conduct two additional tests. First, we extend the holiday analysis by considering holidays in other countries or regions defining the currency involved in the FX rates. As a representative area, Table 5 shows the results for the euro area holidays (the column labeled "EUR"). Looking at the parameter δ_α, it is clear that U.S. holidays have the greatest impact on illiquidity, even though NOK and SEK seem to be affected by European holidays as well; this is perhaps due to geographical proximity and the greater influence of the euro on these regions. Second, in the literature it has been asked whether the asset price variation (Amihud measure's numerator) or transaction volume (Amihud measure's denominator) is priced in stock returns. Similarly, we tested

Table 5
Mispricing (RPVE) vs. liquidity regression estimates. The superscripts *a*, *b*, and *c* indicate significance at 1%, 5%, and 10% level, respectively, based on Newey-West robust standard errors. The columns labeled US and EUR are relative to the dummy based on bank holidays in the United States or the Euro area. The column labeled SNB refers to the dummy variable capturing the SNB shift in currency policy from January 15, 2015.

x=EUR, y=USD	**z=CHF**				**z=GBP**			
	Base	US	EUR	SNB	Base	US	EUR	SNB
α	0.137^a	0.133^a	0.135^a	0.185^a	0.211^a	0.205^a	0.211^a	0.209^a
β	0.097^a	0.200^a	0.197^a	0.139^a	0.073^a	0.154^a	0.147^a	0.148^a
δ_α	–	0.072	0.025	−0.034	–	0.070^b	−0.029	−0.034
δ_β	–	-0.121^a	−0.061	0.116^b	–	-0.108^a	0.011	0.175^b
x=EUR, y=USD	**z=NOK**				**z=SEK**			
	Base	US	EUR	SNB	Base	US	EUR	SNB
α	−0.081	−0.083	−0.092	−0.077	0.090	0.084	0.076	0.093
β	0.304^a	0.613^a	0.623^a	0.599^a	0.186^a	0.380^a	0.389^a	0.363^a
δ_α	–	0.267^c	0.315^b	0.058	–	0.189^b	0.180^a	0.013
δ_β	–	-0.392^b	-0.405^a	0.160	–	-0.278^a	-0.261^a	0.224^c
x=EUR, y=USD	**z=AUD**				**z=CAD**			
	Base	US	EUR	SNB	Base	US	EUR	SNB
α	0.243^a	0.236^a	0.248^a	0.244^a	0.233^a	0.211^a	0.233^a	0.232^a
β	0.074^a	0.157^a	0.143^a	0.144^a	0.055^a	0.135^a	0.109^a	0.110^a
δ_α	–	0.117^a	−0.277	0.402^a	–	0.073^b	−0.043	-0.206^a
δ_β	–	-0.177^a	0.273	-0.192^b	–	-0.131^a	0.061	−0.067
x=EUR, y=USD		**z=JPY**						
	Base	US	EUR	SNB				
α	0.194^a	0.189^a	0.195^a	0.192^a				
β	0.071^a	0.148^a	0.140^a	0.141^a				
δ_α	–	0.060	−0.064	-0.163^c				
δ_β	–	-0.123^a	0.060	0.286^a				

whether our mispricing measure is significantly explained by both factors by regressing $RPVE^{x|y;z}$ on the average RPV and average of $1/\nu$ of $x|z$ and $z|y$ (Lou and Shu, 2017), controlling for their covariance (Amihud and Noh, 2021). We find that for almost all EURUSD triplets, both factors are significantly related to the mispricing measure with the expected sign.[31]

5.2. Shift in currency policy

Another insightful way of studying the relation between liquidity and price efficiency is by analyzing changes in currency policies. Unlike the holidays studied above, these regime changes have lasting effects and, sometimes, they are unexpected. These features apply to a large extent to the announcement of the cap removal of the Swiss franc by the Swiss National Bank (SNB) on January 15, 2015, because this regime change partly surprised the currency markets and led to strong and pervasive effects.[32] Before studying this event, let us briefly summarize the context in which it occurred. Starting in September 6, 2011, the SNB set a minimum exchange rate of 1.20 francs to the euro (capping the franc's appreciation) stating that "the value of the franc is a threat to the economy," and that it was "prepared to buy foreign currency in unlimited quantities" (Swiss National Bank, 2011). To implement this policy, the SNB mainly sold euros to those who demanded Swiss francs with a price below the declared threshold. To do this, the SNB apparently supplied an enormous amount of liquidity.[33] The removal of the declared binding *cap* implied a reduction in the SNB's liquidity supply and trading volume as well as larger fluctuations in Swiss franc rates. All this suggested lower (higher) values of the realized Amihud during (after) the FX capping regime.[34]

Figure 8 provides graphical support for the intuition above. Indeed, daily volatility (RPV) is relatively low until January 15, 2015; it spikes on the day of the announcement of the uncapping, and it remains high until the end of 2016. The CLS trading volume exhibits the opposite behavior; it is relatively high during the capping period and

[31] Furthermore, interacting the two Amihud components with the holiday dummy, as in regression (4), suggests that the negative values of the parameter δ_β are due to the reduction of trading volume rather than volatility. These additional results are available upon request.

[32] Market participants expected that something could happen, but they did not know the exact timing or whether abolishing the peg would result in a new floor, or free floating. Therefore, the SNB announcement was largely unanticipated by market participants (see, e.g., Jermann, 2017; Mirkov et al., 2016). However, the option-based estimation by Hertrich and Zimmermann (2017) indicates at least since mid-November of 2014 an appreciation of the Swiss franc to EUR/CHF 1.15 with a 50% chance within the next few months.

[33] Breedon et al. (2018) provide empirical evidence that the SNB maintained the cap by submitting a large amount of orders with the limit price corresponding to the stated threshold, typically about 500 million euros, in the EBS platform. They also show that this large amount of limit orders was removed seconds before the SNB announcement on January 15, 2015.

[34] This reasoning could be extended to other forms of fixed exchange rate regimes, and the results regarding the Hong Kong dollar in Table 1 are consistent with this idea.

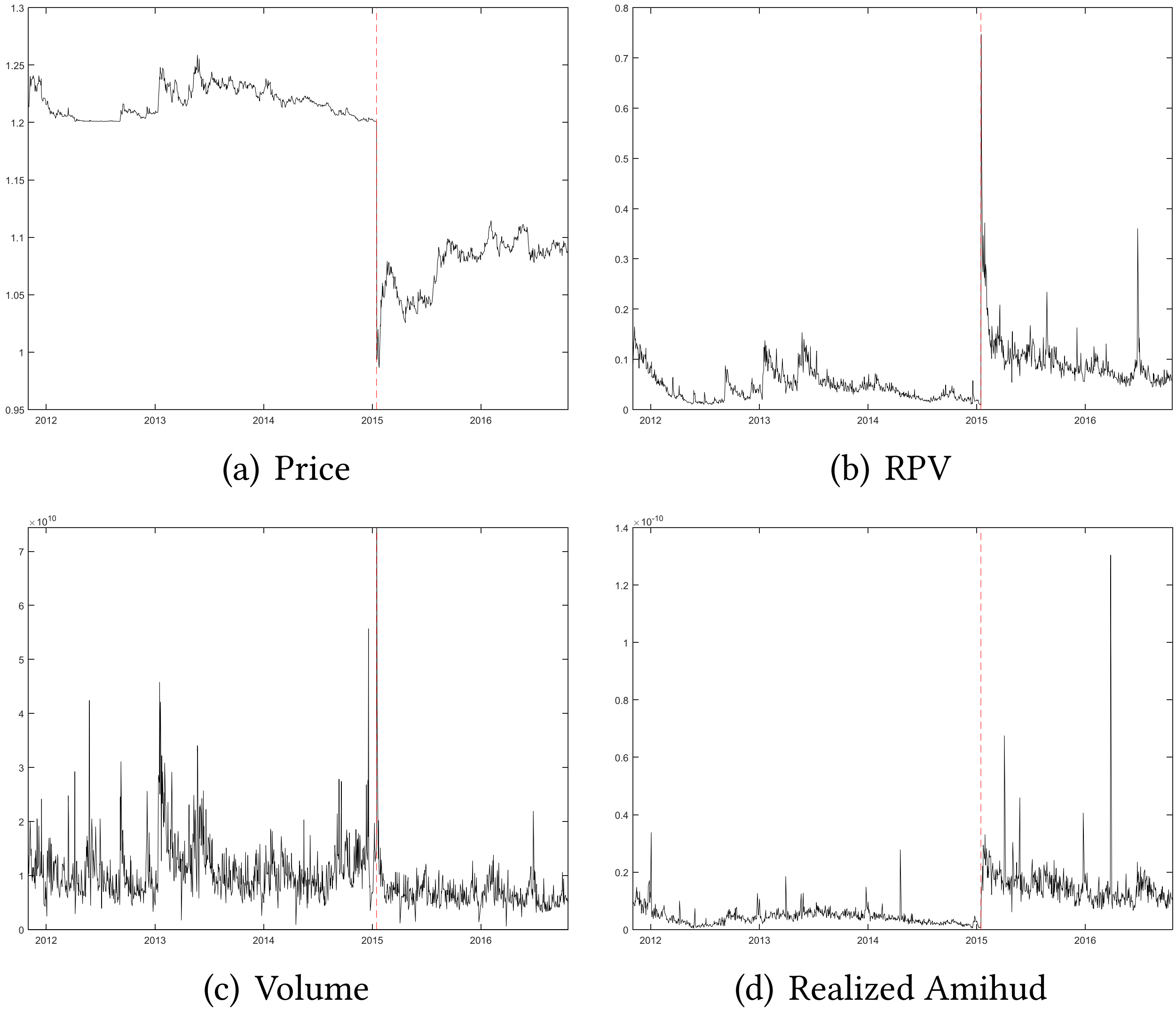


Fig. 8. Spot FX rate of EURCHF from 2012 to 2016 (a), realized power variation (b), trading volume (c) and realized Amihud (d). The announcement date of the cap removal of the Swiss franc by the SNB on January 15, 2015 corresponds to the vertical dashed line.

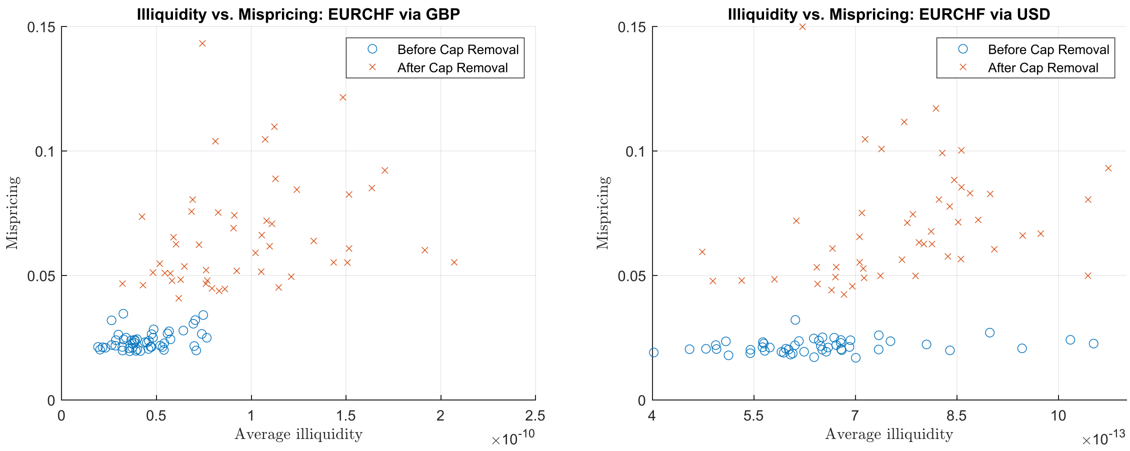
reverts to a much lower value afterwards. Finally, the realized Amihud displays a clear upward shift after the removal of the Swiss franc cap.[35] All in all, the SNB enforcement of its reservation price apparently led to lower volatility, larger trading volume, and higher liquidity. By abandoning this regime, opposite patterns arose.

Fig. 9 illustrates the impact of this currency regime shift on the illiquidity-mispricing relation. Two main effects are observable: (i) a shift of the relation to the right, indicating more illiquidity, and (ii) a shift up, indicating wider mispricing. While the first effect was already observable with expected events such as holidays (see Fig. 7 and Table 5), the second effect suggests that price efficiency can also be altered when major events such as currency regime changes occur. In such cases, the mispricing-illiquidity relation could become stronger. This is confirmed by the estimates of regression (4) reported in the rightmost columns, marked "SNB," of Table 5. In this case, the dummy variable is equal to one when the SNB shifted to the new regime and it replaces the holiday dummy variable for a period of two months after the cap removal. The parameter δ_β is found to be significant and positive, especially for the European currencies, indicating a stronger link between illiquidity and mispricing once the SNB removed the cap.

5.3. Distentangling the mispricing and illiquidity relation

So far, we have considered the average liquidity between the two currency pairs that constitute the legs of the synthetic FX rate ($\overline{A}^{x|y;z}$), which is a sort of measure

[35] Statistical tests for differences in the level of the series in the two subperiods strongly reject the null hypothesis of constant mean in all cases.



(a) GBP (b) USD

Fig. 9. Average illiquidity and mispricing of 50 days prior (blue circles) and after (red crosses) of Swiss franc cap removal on January 15, 2015.

of commonality in market liquidity of the two currency pairs. The results from Eq. (4) show that there is an overall systematic connection between illiquidity and mispricing. However, one could ask how the liquidity of *individual* currencies (e.g., the dollar rather than the euro) contribute to price efficiency. To address this question, we carry out a statistical analysis, performing the following regression

$$RPVE_t^{x|y;z} = \alpha + \beta_1 A_t^{x|z} + \beta_2 A_t^{z|y} + \varepsilon_t, \tag{5}$$

where $A_t^{x|z}$ and $A_t^{z|y}$ are the illiquidity measures on the same "indirect" FX rates used to calculate $RPVE^{x|y;z}$. Since EURUSD is the direct currency pair $x|y$, note that one indirect currency pair is always based on the dollar while the other is based on the euro so as to give us a chance to compare the inelasticity of mispricing to the dollar-based and euro-based liquidity. The first hypothesis is that the illiquidity of both currency pairs contributes to the emergence of price inefficiency. If the trading price impact of both currency pairs ($x|z$ and $z|y$) forming the triangular no-arbitrage with $x|y$ leads to pricing errors, we expect the parameters β_1 and β_2 to be positive and significant. The alternative hypothesis is that only the euro-based currency pair concentrates the illiquidity frictions leading to arbitrage deviations, while traders cluster liquidity in the dollar-based currency pair serving as the vehicle currency that decreases the trading impact (Somogyi, 2021).

Ideally, we would like to conduct robust inference on the elasticity parameter linking better liquidity conditions on $x|z$ and/or $y|z$ to the reduction of the pricing error variability. However, there are at least three reasons to expect that $A_t^{x|z}$ and $A_t^{z|y}$ could correlate with the error term, ε_t, and thereby be endogenous in regression (5), making standard OLS inference on β_1 and β_2 biased. First, there could be omitted variables affecting pricing errors. For example, the transaction costs and idiosyncratic risk (Pontiff, 2016) of each of the three currency pairs might matter in the determination of no-arbitrage violations. Second, there is likely a degree of measurement error in gauging illiquidity by A_t.[36] Third, traders tend to place orders on the most liquid FX rates, rather than on the illiquid ones (where trading is more costly), and the FX liquidity supply is concentrated in a few global dealers who simultaneously act as market makers in multiple currencies. Hence, endogeneity originates by the simultaneous dependence of $A_t^{x|z}$, $A_t^{z|y}$ and $RPVE^{x|y;z}$ on common trading factors. This makes it difficult to establish whether an increase in the pricing errors can be attributed to a deterioration of the liquidity conditions on the base currency x, or on the quote currency, y.

We propose two remedies. First, we include $BAS_t^{x|y}$ and $A_t^{x|y}$, that is, the measures of transaction cost and trading price impact of the direct FX rate $x|y$, as controls to mitigate possible omitted variable bias. Second, we correct for possible endogeneity (stemming from simultaneity in particular) by instrumental variable (IV) techniques, specifically, two-stages least squares (TSLS). The first intuitive idea would be to use the exogeneity represented by bank holidays, in the same fashion as the analysis performed in Fig. 7 and regression (4). However, the binary nature and the relatively limited number of bank holidays render this instrumental variable weak.[37] In the next section, we present a more effective method to address this issue.

5.3.1. Granular instrumental variables

We now illustrate the method of granular instrumental variables by Gabaix and Koijen (2020), as a systematic way to obtain instruments in the context of the relation between triangular FX mispricing and illiquidity. In this con-

[36] Although the superior accuracy of the realized Amihud is proven theoretically and numerically, volatility, as measured by *RPV*, is computed with a finite number of transactions, see the discussion in Meddahi (2002), among others. This leaves open the possibility of an errors-in-variable (EIV) problem in A_t. It leads to a potential attenuation bias towards zero in the estimates of β_1 and β_2 in regression (5). The estimated standard error is biased towards zero, too, so the t-tests are less biased than the parameter estimates, but the EIV problem remains.

[37] See Table 3 in the Internet Appendix.

text, the GIV are created as the difference between the aggregated volume-weighted and equally weighted illiquidity measures on a given base currency x, that is

$$\Gamma_t^x = \sum_{i=1}^{N} w_{i,t} A_t^{x|i} - \sum_{i=1}^{N} e_{i,t} A_t^{x|i}, \tag{6}$$

where $w_{i,t} = \frac{v_t^{x|i}}{\sum_{i=1}^N v_t^{x|i}}$ is the weight based on relative volume, while $e_{i,t} = \frac{1}{N}$ assigns the same weight to the N currencies trading against the base currency.[38] This choice proves to be particularly suited to solving endogeneity issues in a multidimensional context, such as those originating from the trading activity on the global FX market. Let's focus on the currency x (e.g., EUR) and assume that the illiquidity on x is of the form

$$A_t^{x|i} = \mathcal{A}^x w_i (1 + a_t^{x|i}), \tag{7}$$

where $\mathcal{A}^x$ denotes the baseline (unconditional) level of illiquidity on the currency x; w_i is a weight such that $\sum_i w_i = 1$, and $a_t^{x/i}$ is the currency-specific illiquidity shock term, with the following factor structure $a_t^{x/i} = \vartheta_i \eta_t^x + u_{i,t}$, where η_t^x is the illiquidity shock on x common to all currencies; and $u_{i,t}$ is the currency-specific illiquidity shock on x/i that are uncorrelated with ε_t in regression (5). It follows that the total aggregated illiquidity on x is $A_t^x := \sum_i w_i A_t^{x|i} = \mathcal{A}^x w_i (1 + a_{W,t}^x)$. Assuming for simplicity that $\vartheta_i = 1\ \forall i$,[39] we get that $a_t^{x/i} = \eta_t^x + u_{i,t}$, so that $\sum_{i=1}^N w_{i,t} A_t^{x|i} = \eta_t^x + u_{W,t}$, and $\sum_{i=1}^N e_{i,t} A_t^{x|i} = \eta_t^x + u_{E,t}$. Hence, the term $\Gamma_t^x = u_{W,t} - u_{E,t}$ is an idiosyncratic shock to illiquidity on x, and it satisfies the exogeneity condition for an instrumental variable since $E[\epsilon_t \Gamma_t^x] = E[\epsilon_t (u_{W,t} - u_{E,t})] = 0$. In other words, through GIV we break down the simultaneous dependence of illiquidity from the systematic illiquidity components, by extracting idiosyncratic illiquidity shocks from multiple illiquidity measures involving the currency x, disentangling them from the systematic illiquidity component. Hence, we can use GIVs as instruments in Eq. (5) to disentangle the relation between illiquidity and no-arbitrage violations identifying currency-specific patterns.

In the first stage of TSLS, we regress $A_t^{x|z}$ on Γ_t^x and retrieve $\hat{A}_t^{x|z}$, which is the illiquidity on $x|z$ explained by all the aggregated illiquidity shocks on x in the global currency market. Then, $\hat{A}_t^{x|z}$ replaces $A_t^{x|z}$ in Eq. (5), so that the parameter β_1 measures the elasticity of no arbitrage violations that are due to shocks to the illiquidity conditions on x in the global currency markets, ceteris paribus the liquidity conditions of y.[40]

As an illustration, Fig. 10 reports a factor analysis based on the time series of the realized Amihud for the 29 currencies under investigation. By adopting the principal components analysis, we extract the first common factor (in Panel a), which explains around 43% of the total variability of the panel and can be interpreted as a global illiquidity factor. Similarly, we compute equally weighted and volume-weighted global illiquidity factors, and report them in Panel b). The dynamics of the two series are apparently very similar to those of the first PCA factor. However their difference is not negligible. Indeed, by taking a closer look at the weights in Panel a) of Fig. 11, one can clearly see that the weights of the first principal component (black dots) are very close to those of an equally weighted index (red line), while there is much more dispersion when considering weights based on volume in Panel b) of Fig. 11. In particular, the weights of the currency pairs trading against USD are by far the largest ones, providing suggestive evidence of the dollar dominance. This heterogeneity between equally weighted and volume-weighted average across (currency-specific) illiquidity measures is the mechanism beyond the adoption of GIV in this context.

Table 6 reports the parameter estimates of both OLS and TSLS regressions, together with a standard battery of test statistics relevant for the TSLS analysis (Hausman and first-stage F statistic). The Wu-Hausman statistic tests whether instrumentation is needed, that is, whether there is a significant difference between the original (OLS) and instrumented (IV) specifications. As before, we consider EURUSD as the direct currency pair, as it is the most liquid exchange rate and it allows us to obtain the largest number of indirect currencies for the triangular parity, that is, $z = \{AUD, CAD, CHF, GBP, JPY, NOK, SEK\}$.[41]

We adopt three methods: First, the OLS linear regression (5), for which the first column of Table 6 (labeled "OLS") reports the OLS estimated coefficients for each currency. Second, we carry out the GIV estimation and the corresponding estimates are shown in the second column of the same Table (labeled "GIV"). As instruments, we consider GIVs on EUR and USD, denoted Γ_t^{EUR} and Γ_t^{USD}. Third, we extend the GIV approach by including lags of $A_t^{x|z}$ and $A_t^{z|y}$ as further instruments to correct for the EIV problem in the realized Amihud, as suggested by Hansen and Lunde (2014) in the context of volatility measurement (*Lags*). The results of the augmented GIV approach are reported in the third column of Table 6 (labeled "Lags").

Table 6 shows the results for the specifications including the control variables.[42] From an econometric standpoint, the first stage F statistic suggests that the GIV have a strong explanatory power for the endogenous variables in all cases, except AUD. We find that the GIV correction matters, as the Wu-Hausman test rejects the null of consistency of the OLS estimates in all cases, with the exception of AUD. Regarding the parameter estimates of β_{EUR}

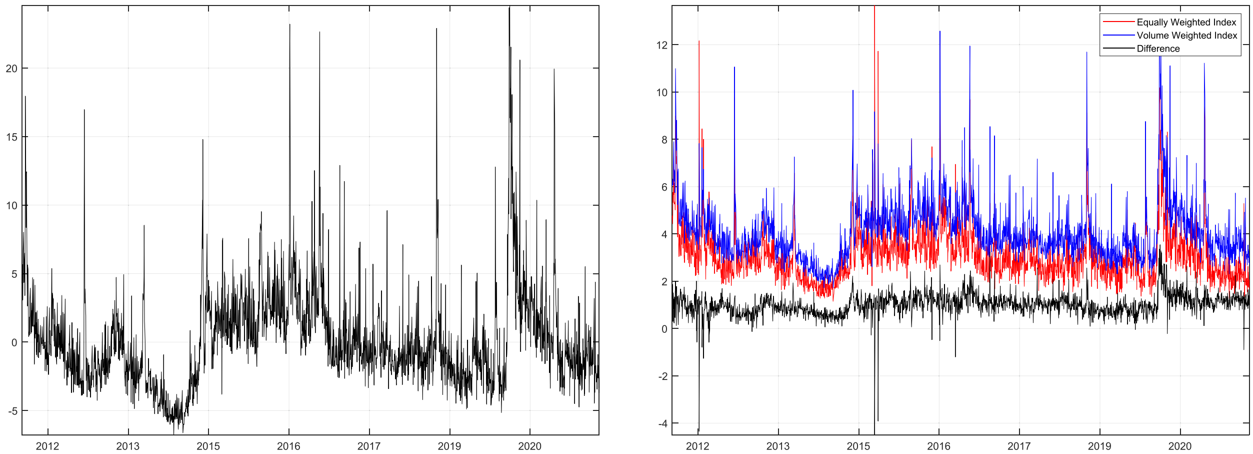
[38] In our sample, we have $N = 14$ currencies trading against USD, and $N = 9$ currencies trading against EUR.

[39] In the empirical analysis below, we show that, as an alternative to the volume-weighting scheme illustrated here, we could extract beforehand the idiosyncratic errors $u_{i,t}^f$ by a factor analysis (for instance, by PCA) and then obtain $\Gamma_t^x = \sum_{i=1}^N (w_{i,t} - e_{i,t}) u_{i,t}^f$.

[40] A GIV for the liquidity conditions on the currency y can be obtained in an analogous way as $\Gamma_t^y = \sum_{i=1}^N w_{i,t} A_t^{y|i} - \sum_{i=1}^N e_{i,t} A_t^{y|i}$. When considering the extended regression in (5) (which includes illiquidity measures involving both x and y), the dependence of $RPVE$ on the illiquidity on x (or y) is partialled-out from the dependence of $RPVE$ on y (or x) by to the Frisch-Waugh-Lovell theorem.

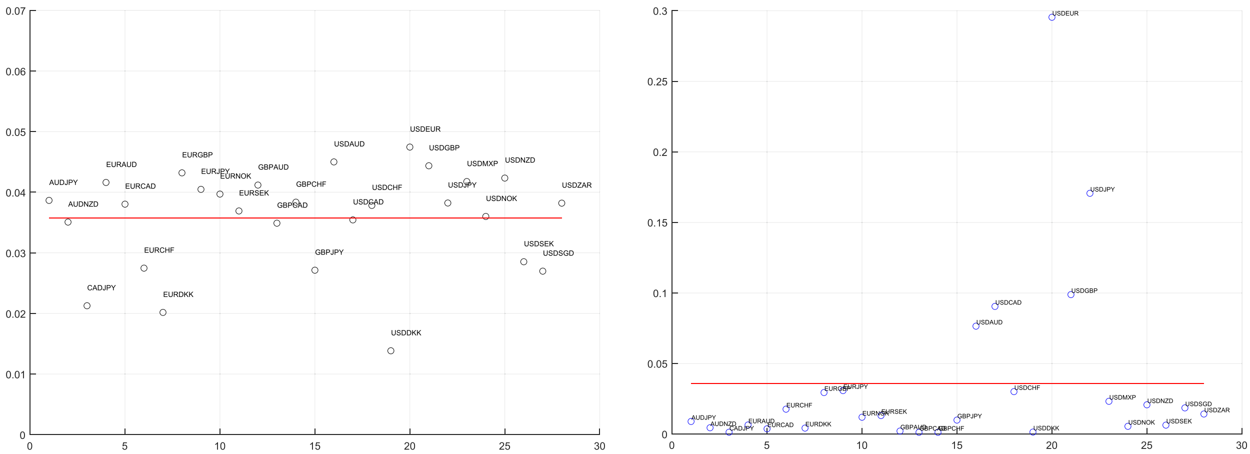
[41] Given that the Danish krone is pegged to the euro, we excluded the analysis of the triplet referring to DKK.

[42] The results excluding the control variables are qualitatively similar and available upon request.



(a) First PCA (b) Equally and volume weighted factors

Fig. 10. Illiquidity factor analysis. Panel a) reports the time series of the global illiquidity factor (black line), that is computed as the first principal component of the panel of 29 currency pairs. The first factor explains 43% of the total variability. Panel b) reports the factor computed using equal weights (in red) and volume-weighted weights (in blue), and their difference (in black).



(a) First factor weights (b) Volume-based weights

Fig. 11. Factor weights. Panel a) displays the weights of the first principal component across the $N = 29$ currency pairs. The red solid line is the reference for the equally weighted index, $w_i = 1/N$. Panel b) displays the weight of the average volume of each currency pair relative to the global average volume. For better visualization, the y-axis of both graphs has been adjusted to the scale of the variables. In Panel a) it ranges from 0 to 0.07, while in Panel b) it ranges from 0 to 0.3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

and β_{USD}, it is worth noting that at least one of the two is positive and significant at the 1% level in all cases and this holds true even when using the (augmented) GIV methods.

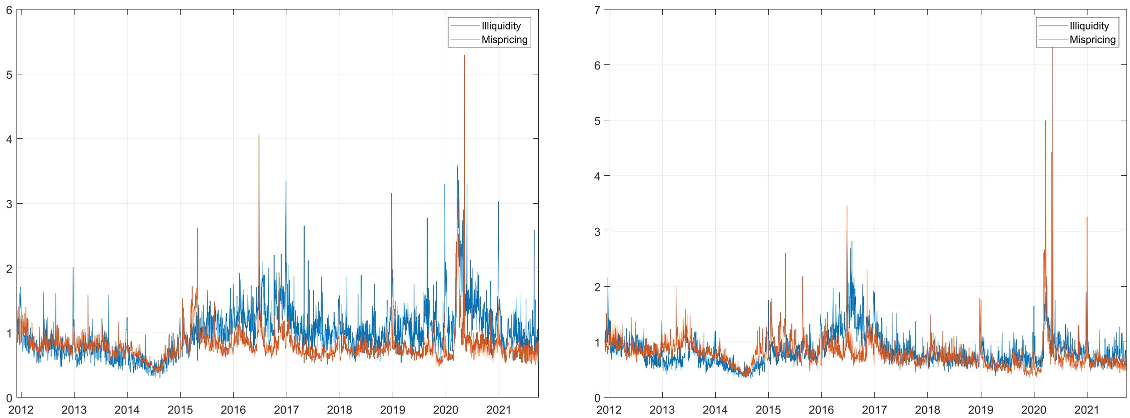
From an economic standpoint, the main message that emerges is that the illiquidity of the currency pairs involving the euro rather than those based on the dollar mostly contribute to mispricing. In other words, β_{EUR} is most of the time positive and significant supporting the (alternative) hypothesis that the pairs involving the euro, which are typically traded in thinner markets, concentrate the illiquidity frictions linked to deviations from the triangular no-arbitrage parity. In contrast, the dollar pairs serving as vehicle currencies are mostly disconnected from mispricing. Overall, the dollar liquidity appears crucial for price efficiency, while arbitrage deviations are more connected to euro illiquidity, reinforcing the idea that the dollar plays a predominant role in determining the liquidity conditions in the global FX market and providing suggestive evidence that dollar liquidity begets FX price efficiency.

5.4. Long-term equilibrium between mispricing and illiquidity

The analyses performed above have clearly shown that liquidity and price efficiency are interconnected, and that this relation varies across time, currencies, and regimes. This is also confirmed by the visual inspection of Fig. 12, which shows a very kindred movement of illiquidity and

Table 6
Mispricing (RPVE) vs. illiquidity regression estimates. The superscripts *a*, *b*., and *c* indicate significance at the 1%, 5%, and 10% level, respectively, based on Newey-West robust standard errors. The regression model includes the illiquidity measure on the direct FX rate, $A^{x|y}$, as well as the relative BAS on the direct rate, as control variables.

x=EUR, y=USD	z=CHF			z=GBP			z=NOK			z=SEK		
	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags
α	−0.054	−0.055	−0.063^{c}	−0.017	−0.017	−0.03	−0.243^{b}	−0.320^{b}	−0.314^{a}	−0.130^{b}	−0.153^{a}	−0.158^{a}
β_{EUR}	0.092^{a}	0.196^{a}	0.104^{a}	0.074^{a}	0.145^{a}	0.122^{a}	0.558^{a}	0.637^{a}	0.707^{a}	0.363^{a}	0.470^{a}	0.491^{a}
β_{USD}	0.146	0.202	0.052	0.051^{a}	0.026	0.091^{a}	−0.066^{c}	−0.284^{a}	−0.329^{a}	−0.078^{a}	−0.168^{a}	−0.188^{a}
R^2	0.252	0.252	0.253	0.562	0.567	0.578	0.597	0.605	0.61	0.527	0.542	0.56
First-Stage F	–	17.67	222.5	–	189.6	186.7	–	391.3	218.7	–	618.7	403.9
Hausman test	–	7.388^{b}	2.214	–	9.344^{b}	31.93^{a}	–	20.87^{a}	30.43^{a}	–	42.03^{a}	64.05^{a}
x=EUR, y=USD	**z=AUD**			**z=CAD**			**z=JPY**					
	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags			
α	−0.121^{a}	−0.112^{a}	−0.134^{a}	−0.025^{c}	−0.021	−0.044^{b}	−0.059^{b}	−0.057^{b}	−0.062^{b}			
β_{EUR}	0.082^{a}	0.128^{a}	0.048^{b}	−0.012^{b}	0.066^{a}	−0.009^{c}	0.056^{a}	0.149^{a}	0.066^{a}			
β_{USD}	−0.008	−0.075^{a}	0.087^{a}	0.024^{a}	0.082^{a}	0.192^{a}	−0.022	−0.055^{a}	−0.016			
R^2	0.573	0.572	0.578	0.654	0.674	0.663	0.561	0.568	0.561			
First-Stage F	–	2.328	236.9	–	76.47	23.14	–	61.00	348.5			
Hausman test	–	1.805	23.98^{a}	–	16.56^{a}	121.07^{a}	–	11.62^{a}	4.95^{c}			



(a) GBP (b) JPY

Fig. 12. Time series of illiquidity and pricing errors. The figure reports the time series of illiquidity based on the average realized Amihud (blue line, $\bar{A}_t^{EUR|USD;z}$) and mispricing (red line, $RPVE_t^{EUR|USD;z}$) of EURUSD when considering triangular no-arbitrage relations with $z = GBP$ (Panel a) and $z = JPY$ (Panel b). The time series are indexed to the first observation in the sample for better visualization.

mispricing across years. The next natural step is therefore to ask how their connection exactly evolves dynamically and across currencies. To shed light on this issue, we need to consider a proper econometric framework able to establish (i) if the two series revert in the long run to a steady-state equilibrium and (ii) if and for how long a positive shock on illiquidity affects mispricing and vice versa.

An appropriate setting in which to do this is to study the equilibrium between mispricing and illiquidity in a bivariate system allowing for cointegration. More specifically, we examine the mispricing-illiquidity dynamic relation by means of the fractional vector error-correction model (FVECM) of Granger (1986) (see also Johansen, 2008),

$$\Delta^d Y_t = \alpha\beta' L_b \Delta^{d-b} Y_t + \sum_{j=1}^{k} \Gamma_j \Delta^d Y_{t-j} + \varepsilon_t \,, \tag{8}$$

in which $Y_t = (RPVE_t^{x|y;z}, \bar{A}_t^{x|y;z})'$ contains the time series of illiquidity and mispricing, and $\varepsilon_t \sim N(0, \Sigma)$. In (8), the coefficient $d \geq 0$ determines the order of (fractional) integration of the series Y_t: the larger the d, the more persistent are the effects of shocks on the dynamics of the series, a feature called *long-memory*.[43] The term Δ^d denotes the *fractional* difference operator, $\Delta^d := (1-L)^d = \sum_{j=0}^{\infty}(-1)^j\binom{d}{j}L^j$, with L the (ordinary) lag operator, that is, $LY_t = Y_{t-1}$. The parameter b denotes the *cointegration gap* and it is such that $0 \leq b \leq d$, while $L_b := 1 - \Delta^b$ is the *fractional lag operator*. In other words, while Y_t is a process in-

[43] When $d = 0$, Y_t evolves as a weakly stationary process (such as a white noise), while when $d = 1$, Y_t evolves as a random walk. In the FVECM, the parameter d can take fractional values (typically in the range between 0 and 1), providing great flexibility in describing the dynamics of persistent processes.

Table 7
Cointegration analysis. Parameter estimates for the FVECM$_d$ model. The table reports the estimates and standard errors of the long-memory parameter d, the speed of reversion coefficients (α_{RPVE} and α_{Amihud}). The table also reports the number of selected lags in the short-memory term (k) and the value of the trace test for the cointegration rank being equal to 1 (with associated p-value in parentheses). The time series of $RPVE_t^{EUR|USD;z}$ and $\bar{A}_t^{EUR|USD;z}$ are both indexed to the first observation in the sample, to ease the interpretation of the cointegration coefficient β_2.

	Cointegration Analysis													
x=EUR, y=USD	**z=SEK**		**z=NOK**		**z=JPY**		**z=GBP**		**z=AUD**		**z=CAD**		**z=CHF**	
	Est.	Std.E.	Est.	Std.E.	Est.	Std.E.	Est.	Std.E.	Est.	Std.E.	Est.	Std.E.	Est.	Std.E.
d	0.638[a]	0.012	0.657[a]	0.036	0.662[a]	0.018	0.832[a]	0.034	0.676[a]	0.021	0.741[a]	0.022	0.628[a]	0.021
b	0.638[a]	0.012	0.657[a]	0.036	0.662[a]	0.018	0.832[a]	0.034	0.676[a]	0.021	0.741[a]	0.022	0.628[a]	0.021
α_{RPVE}	−0.026	0.024	0.029	0.029	−0.008	0.016	−0.002	0.014	0.016	0.018	−0.033	0.017	0.032	0.016
α_{Amihud}	0.147[a]	0.025	0.176[a]	0.024	0.091[a]	0.015	0.161[a]	0.024	0.121[a]	0.023	0.341[a]	0.033	0.150[a]	0.022
β_2	0.927	–	1.002	–	0.872	–	0.776	–	0.486	–	0.901	–	0.704	–
lags (k)	2	–	4	–	2	–	4	–	3	–	3	–	2	–
$rank = 1$	1.449	(0.217)	0.552	(0.432)	2.654	(0.113)	0.122	(0.687)	1.424	(0.217)	0.905	(0.321)	1.314	(0.234)

tegrated of order d, or $I(d)$, the departures from equilibrium (or error-correction term), that is, $\beta' Y_t$, are $I(d-b)$. Note that the model in (8) is more general than the ordinary VECM model, studied in Johansen (1991, 1995), which corresponds to the special case $d = b = 1$. The long-run equilibrium dynamics in (8) are governed by the $2 \times r$ matrices α and β, where α determines the speeds of error correction or adjustment to equilibrium, and β contains the cointegration vectors representative of the equilibrium relations between $RPVE_t^{x|y;z}$ and $\bar{A}_t^{x|y;z}$. In the current bivariate system, the cointegration rank r is such that $r \in \{0, 1, 2\}$ and long-run equilibria (or cointegration) among $RPVE_t^{x|y;z}$ and $\bar{A}_t^{x|y;z}$ are found only if $r = 1$. In this case, it is standard practice to normalize the cointegration vector β as $\beta = (1, -\beta_2)'$, so that $\beta' Y_t = RPVE_t^{x|y;z} - \beta_2 \bar{A}_t^{x|y;z}$ is the error-correction term. The unknown equilibrium parameter β_2 has to be estimated by maximum likelihood alongside the other parameters of the model, including d and b; see Johansen and Nielsen (2012). The short-run dynamics are governed by the 2×2 matrices Γ_j, $j = 1, \ldots, k$, and $\Sigma > 0$ is the positive definite covariance matrix of the FVECM error terms ε_t.

The parameter estimates of model (8) are reported in Table 7. As before, we consider triangular arbitrage for EURUSD against the same set of currencies (AUD, CAD, CHF, JPY, GBP, NOK, and SEK). Three outcomes stand out. First, in all cases we cannot reject the null hypothesis that the cointegration rank is 1 (p-values are above 10% in all cases), corroborating the idea that illiquidty and mispricing are actually tied together in the long run. The estimates of d are in the range between 0.5 and 1, meaning that RPVE (mispricing) and realized Amihud (illiquidity) are persistent but mean reverting processes, as opposed to the random walk ($d = 1$), which is a non-mean-reverting process. Furthermore, the estimates b are equal to those of d (thus $d - b = 0$), meaning that the estimated error correction term ($\hat{\beta}' Y_t$) is a non persistent process. In other words, shocks to the equilibrium relation between illiquidity and mispricing are short lived. Second, the estimate of β_2 is positive and close to one for all currencies (except AUD), suggesting that a change in $RPVE_t^{x|y;z}$ is associated with a corresponding one-to-one change in $\bar{A}_t^{x|y;z}$. Third, the estimates of the parameters α reveal which variable between RPVE and realized Amihud moves to restore the equilibrium when a shock hits the system. Notably, only the estimates of α for the realized Amihud are significant, suggesting that, when a deviation to the equilibrium occurs (that is $\beta' Y_t \neq 0$), it is rather a liquidity improvement that restores the equilibrium. The most liquid triplet, that is the one involving the Japanese yen, reports the lowest estimated α in line with the result above, indicating that the level of liquidity is important to maintain high price efficiency.

6. Conclusion

In this paper, we analyzed the liquidity of the global currency market. In addition to transaction costs that have already been considered in previous literature, we studied another important dimension of market illiquidity, that is, how trading volume impacts currency prices. To do this, we proposed a refinement of the popular Amihud (2002) illiquidity measure that can be interpreted as the amount of FX volatility associated with a unit of trading volume, which decreases with market depth and the elasticity of the demand curve for currencies.

To study the liquidity of the global currency market, we gained access to an intraday data set of FX trading volume from CLS Group, including 29 currency pairs spanning November 2011 to September 2021. We enhanced the original Amihud measure by using high-frequency return variations rather than the daily absolute return. In doing so, we gained a more accurate measurement of return volatility and a more precise estimate of price impact.

The time-series analyses show that trading impact increases when the currency market is thinner, which regularly occurs outside “London hours’ and when there are bank holidays in major financial centers. Furthermore, price impact increases with transaction costs, money market strains, uncertainty, and risk aversion. Cross-sectionally, we quantify the price impact of trading individual currency pairs, and show that those involving U.S. dollars enable traders to avoid more adverse price impacts.

In the second part of our paper, we analyzed the relation between liquidity and price efficiency by looking at violations of the “triangular” no-arbitrage condition. We find that mispricing systematically increases with illiquidity, suggesting that a large price impact limits arbitrage.

Currencies benefiting from an average smaller trading impact are able to maintain a higher level of price efficiency. Furthermore, currency pairs involving the euro, which are typically traded in thinner markets, concentrate the illiquidity frictions leading to arbitrage deviations. This does not apply to the dollar pairs serving as vehicle currencies, supporting the idea that it is rather the dollar liquidity that begets price efficiency measured as deviations from the law of one price.

Appendix A. Theoretical framework

Let us consider a world with two currencies, x (base) and y (quote). We assume that the market consists of a finite number $J \geq 2$ of active participants, who trade on the FX rate $x|y$. Within a given trading period of unit length (e.g, one hour, day, or week), the market for the currency pair $x|y$ passes through a sequence of $i = 1, \ldots, I$ equilibria. The evolution of the equilibrium price is motivated by the arrival of new information to the market. At intra-period i, the desired position of the jth trader ($j = 1, \ldots, J$) on the FX rate $x|y$ is given by

$$q_{i,j}^{x|y} = \xi^{x|y}(p_{i,j}^{x|y;*} - p_i^{x|y}), \quad \xi^{x|y} > 0, \tag{A.1}$$

where $p_{i,j}^{x|y;*}$ is the reservation price of the jth trader and $p_i^{x|y}$ is the current market price (both measured in logs). The equilibrium function in (A.1) is analogous to the theory of Clark (1973), and Tauchen and Pitts (1983), which provides an abstract representation of the supply/demand mechanism on the market. Although this is not the conventional microstructure theoretical framework and does not offer the straightforward interpretation of the price impact due to asymmetric information in the spirit of Kyle's model (Chordia et al., 2009), our theoretical framework has two features: First, the reservation price of each trader might broadly reflect some of the following aspects: individual preferences, liquidity issues, asymmetries in information sets, and different expectations about the fundamental values of the FX rate. In general, the reservation price can deviate from the market price for an idiosyncratic reason, inducing the jth trader to trade. Second, it allows us to build a theory for the formation of the trading volume (rather than the order flow), which appears at the denominator of the realized Amihud index.

The term $\xi^{x|y}$ is a positive constant capturing the market depth: The larger the $\xi^{x|y}$, the larger the quantities of x that can be exchanged for y (and vice versa) for a given difference $p_{i,j}^{x|y;*} - p_i^{x|y}$. In other words, $\xi^{x|y}$ measures the capacity of the market to allow large quantities to be exchanged at the intersection between demand and supply, thus recalling the concept of market depth and resilience that reduces the price impact of trading. If $(p_{i,j}^{x|y;*} - p_i^{x|y}) > 0$, the jth trader believes that the equilibrium trading price of $x|y$ is too low, that is, currency x should be more expensive relatively to y. Therefore, she will buy x and sell y. On the contrary, if $(p_{i,j}^{x|y;*} - p_i^{x|y}) < 0$, the jth trader will buy y and sell x. The quantity exchanged for a unit change of $p_{i,j}^{x|y;*} - p_i^{x|y}$ is given by the slope $\xi^{x|y}$.

By market clearing, that is, $\sum_j q_{i,j}^{x|y} = 0$, we have that the average of the reservation prices clears the market, that is, $p_i^{x|y} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|y;*}$, and the log-return is $r_i^{x|y} := \Delta p_i^{x|y} = p_i^{x|y} - p_{i-1}^{x|y}$. Furthermore, as new information arrives, the traders adjust their reservation prices $\Delta p_{i,j}^{x|y;*}$, resulting in a change in the market price given by the average of the increments of the reservation prices. Consequently, the generated trading volume in each ith sub-interval is

$$\nu_i^{x|y} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J} |\Delta p_{i,j}^{x|y;*} - \Delta p_i^{x|y}|,$$

where $\Delta p_{i,j}^{x|y;*} = p_{i,j}^{x|y;*} - p_{i-1,j}^{x|y;*}$. We assume the following dynamics for the reservation price about the $x|y$ FX rate

$$dp_j^{x|y;*}(t) = \mu_j^{x|y}(t)dt + \sigma_j^{x|y}(t)dW_j^{x|y}(t), \quad j = 1, \ldots, J \tag{A.2}$$

where $\{W_j(t), j = 1, \ldots, J\}$ is a collection of independent Wiener processes. The term $\sigma_j^{x|y}(t) \geq 0$ is the stochastic volatility process of the jth trader, and we assume it to be locally square integrable. The term $\mu_j(t)$ is a predictable process with finite variation, which might represent the long-run expectation of the jth trader about the FX rate and could be a function of fundamental quantities such as interest rates differentials and long-term macroeconomic views.

By letting $\sigma_j^{x|y}$ vary across traders, we introduce heterogeneity among them. This reconciles with many realistic features including the evidence of long-memory in volatility, obtained by the superposition of traders operating at different frequencies; see, for instance, the heterogeneous autoregressive models of Müller et al. (1997) and Corsi (2009). This setup is consistent with a representation of a frictionless market in which each trader participates through its reservation price to the determination of a new equilibrium price by carrying new information. For ease of exposition and tractability, we assume that trades happen on an equally spaced and uniform grid, $i = 1, 2, \ldots, I$. On the ith discrete sub-interval of length $\delta = \frac{1}{I}$, we have that

$$p_{i,j}^{x|y;*} = \int_{\delta(i-1)}^{\delta i} \mu_j^{x|y}(s)ds + \int_{\delta(i-1)}^{\delta i} \sigma_j^{x|y}(s)dW_j^{x|y}(s). \tag{A.3}$$

Based on the return on the ith interval, we can consider the realized variance, defined as $RV^{x|y} = \sum_{i=1}^{I}(r_i^{x|y})^2$ with $\delta = 1/I > 0$, as introduced by Andersen and Bollerslev (1998). Following Barndorff-Nielsen and Shephard (2002a,b), taking the limit for $\delta \to 0$ (that is, $I \to \infty$), we get $p\lim_{I\to\infty} RV^{x|y} = \frac{1}{J^2}\mathcal{V}_{x|y}$, where $\mathcal{V}_{x|y} = \sum_{j=1}^{J}\mathcal{V}_{x|y,j}$ is the variation of the FX rate on the unit interval generated by the aggregated individual components of $r^{x|y}$. The term $\mathcal{V}_{x|y,j} = \int_0^1 \left(\sigma_j^{x|y}(s)\right)^2 ds$ is the *integrated variance* associated with the jth trader's specific component. The term $\mu_j(t)$ does not enter into the expression of $V_{x|y,j}$ since the magnitude of the drift, when measured over infinitesimal intervals, is dominated by the diffusive component of $\Delta p_j^{x|y,*}(t)$ driven by the Brownian motion.

Following Barndorff-Nielsen and Shephard (2003), for a given $\delta > 0$ we can also define the *realized power variation* of order one (or realized absolute variation) as $RPV^{x|y} = \sum_{i=1}^{I} |r_i|$. Hence, analogously to the illiquidity proxy in Amihud (2002), we can define a new version of the Amihud illiquidity measure, namely the realized Amihud, as

$$A^{x|y} := \frac{RPV^{x|y}}{\nu^{x|y}}. \tag{A.4}$$

This quantity gauges the price impact of trading, that is, the amount of volatility on a unit interval (as measured by $RPV^{x|y}$) associated with the trading "dollar" volume $\nu^{x|y} = \sum_{i=1}^{I} \nu_i^{x|y}$ generated in the same period. In other words, $A^{x|y}$ measures the amount of FX volatility associated with a unit of trading volume. The following proposition highlights the main determinants of this illiquidity measure.

Proposition 1. Consider the illiquidity measure defined in (1). As $I \to \infty$ and under homogeneity of traders, that is, $\sigma_j^{x|y} = \sigma^{x|y} \quad \forall j = 1, 2, \ldots, J$,

$$p\lim_{I\to\infty} A^{x|y} = \frac{2}{\xi^{x|y} J\sqrt{(J-1)}}. \tag{A.5}$$

The proof follows by applying the properties of the super-position of independent SV processes;[44] the limit for $\delta \to 0$ (or $I \to \infty$) of $RPV^{x|y}$ (scaled by $\delta^{1/2}$ to guarantee a nondivergent sequence) is

$$p\lim_{I\to\infty} \delta^{1/2} RPV^{x|y} = \sqrt{\frac{2}{\pi}} \mathcal{S}_{x|y}, \tag{A.6}$$

where $\mathcal{S}_{x|y} = \int_0^1 \overline{\sigma}^{x|y}(s) ds$ is the integrated average standard deviation, where the latter is defined as $\overline{\sigma}^{x|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} \sigma_j{}^{x|y^2}(t)}$. The aggregate volume of $x|y$ on a unit (daily) interval is $\nu^{x|y} = \sum_{i=1}^{I} \nu_i^{x|y}$, and letting $I \to \infty$, we get (considering again the same scaling by $\delta^{1/2}$ to guarantee a nondivergent sequence)

$$p\lim_{I\to\infty} \delta^{1/2} \nu^{x|y} = \frac{\xi^{x|y}}{2}\sqrt{\frac{2}{\pi}} \overline{\mathcal{S}}_{x|y}, \tag{A.7}$$

with $\overline{\mathcal{S}}_{x|y} = \frac{1}{J}\sum_{j=1}^{J}\int_0^1 \varsigma_j^{x|y}(s) ds$, where $\varsigma_j^{x|y}(t) = \sqrt{(J-1)^2 \sigma_j{}^{x|y^2}(t) + \sum_{s\neq j} \sigma_s{}^{x|y^2}(t)}$.

Hence, we get that

$$p\lim_{I\to\infty} A^{x|y} = \frac{2\mathcal{S}_{x|y}}{\xi^{x|y}\overline{\mathcal{S}}_{x|y}}, \tag{A.8}$$

which reflects the ratio of the total average standard deviation carried by each trader. Under homogeneity of the traders, that is, $\sigma_j^{x|y}(t) = \sigma^{x|y}(t)$, $\varsigma_j^{x|y}(t)$ reduces to $\sqrt{J(J-1)}\sigma^{x|y^2}(t)$, and we get that

$$\overline{\mathcal{S}}_{x|y} = J\sqrt{J-1}\mathcal{S}_{x|y}, \tag{A.9}$$

so that Proposition 1 follows directly.

Proposition 1 shows that on a period of unit length (an hour, day, or month), $A^{x|y}$ is inversely related to the slope $\xi^{x|y}$ of the equilibrium function in (A.1). That is, for a given difference between the reservation price and the market price, $A^{x|y}$ decreases as this slope increases. For large values of $\xi^{x|y}$, large volume would be associated with small variations between the prevailing price and the reservation price for each trader, thus signaling market depth and liquidity. Instead, when $\xi^{x|y} \to 0^+$, that is, in the limiting case of a flat equilibrium function in (A.1), the liquidity is minimal (and $A^{x|y}$ diverges) since no actual trade takes place. Under the assumption of homogeneity of the traders, $\sigma_j^2(t) = \sigma^2(t) \;\; \forall j = 1, \ldots, J$, Proposition 1 also highlights the inverse relation between the number of active traders and illiquidity.[45]

The baseline assumptions of the model (linearity of the trading function, constant number of active traders, and absence of frictions) are inevitably abstract. As for the form of the equilibrium function in (A.1), note that the trades take place on short intraday intervals of length $\delta = 1/I$ and are generally associated with small price variations. Therefore, it is not restrictive to assume the equilibrium function to be linear on small price changes and a fixed number of traders during such a short period. In particular, the assumption that J traders observe the same market price could be reconciled with a reference price provided by an electronic limit order book platform such as EBS, Refinitiv FX Matching, or Hotspot FX ECN. Finally, illiquidity frictions could be introduced by combining the theory of Tauchen and Pitts (1983) with that of the demand of immediacy by Grossman and Miller (1988); see Darolles et al. (2015, 2017).[46] The next step is to verify with a numerical analysis the theoretical predictions and robustness of the realized Amihud to microstructural frictions. Then, we will show whether this theory can be successfully adopted to characterize illiquidity and mispricing in the global FX market.

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[44] As in the case in Barndorff-Nielsen and Shephard (2002b), $\overline{\Delta p_i}^{x|y,*}(t) = \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{x|y,*}$ is equivalent in law to $\overline{\Delta p_i}^{x|y,*} = \int_{\delta(i-1)}^{\delta i} \overline{\sigma}^{x|y}(t) dW^{x|y,*}(t)$, where $\overline{\sigma}^{x|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} \sigma_j{}^{x|y^2}(t)}$.

[45] Relaxing the assumption of homogeneity would result in a ratio of two aggregated volatility measures, each estimating the weighted average of the variance carried by each trader; see Eq. (A.8).

[46] The latter propose a parametric filtering technique to achieve a dynamic decomposition of trading volume into a component associated with other dimensions of illiquidity (e.g., the asymmetric information between informed vs. uninformed traders), and a part associated with the efficient price variations as in Eq. (A.3), which are a function of the latent information flow. The methodology of Darolles et al. (2017) could be employed to achieve a refined version of the realized Amihud robust to the possible contamination arising from other liquidity frictions. This is beyond the scope of the present analysis and it is left to future research.

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